

Chang Learning SAT Lesson 8: Graphing Linear & Quadratic Functions

Linear Equations

The SAT mathematics sections will contain several graphing problems. Most of the questions are with functions, which are also either linear or quadratic. A few other functions are also tested: cubic functions, square root, exponential equations, logarithms, exponential growth & decay, and some other investing formulas. This lesson will focus on the graphing problems related to functions with linear and quadratic equations.

Linear Equation

Slope Intercept Form: $y = mx + b$

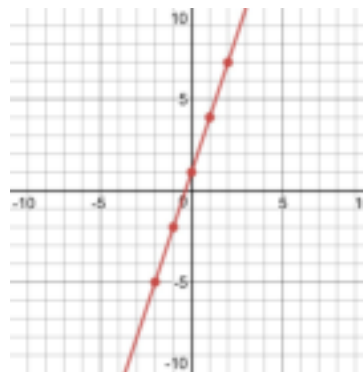
$$m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$$

$b = y - \text{intercept}$

The equation shown is
 $y = 3x + 1$

$$m = \text{slope} = 3$$

$$b = y - \text{intercept} = 1$$



Linear Equation

Standard Form: $ax + by = c$

Use an intercept table to find both the x and y intercepts.

$$3x + 2y = 6$$

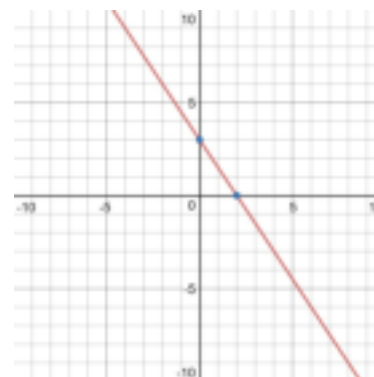
When $x = 0$, $3(0) + 2y = 6$. Therefore $y = 3$.

When $y = 0$, $3x + 2(0) = 6$. Therefore $x = 2$.

Intercept Table

| x | y |
|---|---|
| 0 | 3 |
| 2 | 0 |

Intercepts
x-intercept (2,0)
y-intercept (0,3)



For several questions on the SAT, it is useful to know how to “isolate a variable” and change the equation from one form to another in various problems.

1) Given $3x - 2y = 12$, what is the equation in slope intercept form ?

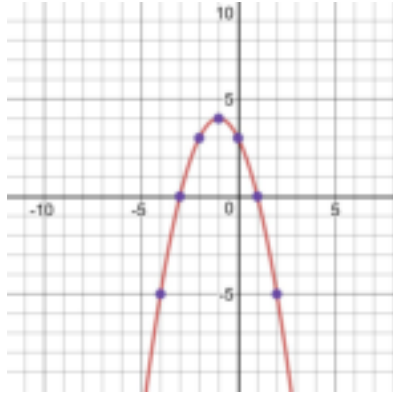
- A) $y = \frac{2}{3}x - 6$
- B) $y = -\frac{2}{3}x + 6$
- C) $y = \frac{2}{3}x - 12$
- D) $y = -\frac{2}{3}x + 12$
- E) $y = -\frac{3}{2}x + 12$

Subtract $3x$ from both sides, then divide both sides by -2 . Choice A is correct.

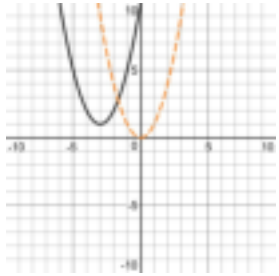
Chang Learning SAT Lesson 8: Graphing Linear & Quadratic Functions

Quadratic Equations

When plotting quadratic equations, knowing the location of the vertex, and if the parabola opens up or down, is obviously paramount. One technique of graphing these forms is to use the “7 point parabola”. Plot the vertex, then 3 points before and after the vertex equally spaced. The natural symmetry of this curve will reveal the 3 points before are mapped to the same dependent values as the points after.

| <p>Standard Form: $y = ax^2 + bx + c$</p> <p>$a > 0$, the parabola opens up $a < 0$, the parabola opens down</p> <p style="text-align: center;">Vertex (x,y) The maximum or minimum</p> <p style="text-align: center;">Located at $x = \frac{-b}{2a}$, called the axis of symmetry.</p> | <p>7 Point Parabola</p> <table border="1" style="margin: auto; border-collapse: collapse;"> <thead> <tr> <th style="width: 10%;"></th> <th style="width: 10%;">X</th> <th style="width: 10%;">Y</th> </tr> </thead> <tbody> <tr><td></td><td>-4</td><td>-5</td></tr> <tr><td></td><td>-3</td><td>0</td></tr> <tr><td></td><td>-2</td><td>3</td></tr> <tr><td>Vertex</td><td>-1</td><td>4</td></tr> <tr><td></td><td>0</td><td>3</td></tr> <tr><td></td><td>1</td><td>0</td></tr> <tr><td></td><td>2</td><td>-5</td></tr> </tbody> </table> | | X | Y | | -4 | -5 | | -3 | 0 | | -2 | 3 | Vertex | -1 | 4 | | 0 | 3 | | 1 | 0 | | 2 | -5 | <p>$y = -x^2 - 2x + 3$</p>  |
|---|--|----|---|---|--|----|----|--|----|---|--|----|---|---------------|----|---|--|---|---|--|---|---|--|---|----|--|
| | X | Y | | | | | | | | | | | | | | | | | | | | | | | | |
| | -4 | -5 | | | | | | | | | | | | | | | | | | | | | | | | |
| | -3 | 0 | | | | | | | | | | | | | | | | | | | | | | | | |
| | -2 | 3 | | | | | | | | | | | | | | | | | | | | | | | | |
| Vertex | -1 | 4 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 0 | 3 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | 0 | | | | | | | | | | | | | | | | | | | | | | | | |
| | 2 | -5 | | | | | | | | | | | | | | | | | | | | | | | | |

Although most of the SAT questions are commonly in standard form, the past few years there has been an increase in the presence of Vertex Form of a Parabola. A good SAT practice before the exam is to work through a few problems with the general equation $y = a(x - h)^2 + k$ which has vertex point (h, k) .

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| <p>Vertex Form: $y = a(x - h)^2 + k$</p> <p>$a > 0$, the parabola opens up $a < 0$, the parabola opens down</p> <p style="text-align: center;">The Vertex (x,y) point, the maximum or minimum is located at $(x, y) = (h, k)$</p> |  |
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Vertex form is useful for transformation. The original parabola $y = x^2$ is shown in the graph. The parabola has been moved 3 units left and one unit up. The new equation for the parabola is $y = (x + 3)^2 + 1$. By using FOIL, this is equivalent in standard form to $y = x^2 + 6x + 10$.

Chang Learning SAT Lesson 8: Graphing Linear & Quadratic Functions

SAT Lesson #8 Classwork: Quadratic Formulas

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| 1) Which of the following points is closest to the point $(x,y) = (2,3)$? | A) (2,2) B) (2,3.5) C) (2.5,2.5) D) (1,2) |
| 2) What is the area of square ABCD with opposite corners at $A(5,7)$ and $C(-2,0)$? | A) 25 B) 36 C) 49 D) 81 |
| 3) A lattice point has coordinates (x,y) that are both integers. How many lattice points are contained in $x^2 + y^2 \leq 6$, the circle centered at the origin with radius $\sqrt{6}$? | A) 15 B) 17 C) 19 D) 21 |
| 4) Which line is parallel to $y = 3x + 1$? | A) $y = 3x - 5$ B) $y = -3x + 2$ C) $y = \frac{1}{3}x + 7$ D) $y = -\frac{1}{3}x + 1$ |
| 5) Which line is perpendicular to $y = -2x + 4$? | A) $y = 2x + 5$ B) $y = 4x - 2$ C) $y = -\frac{1}{2}x + 1$ D) $y = \frac{1}{2}x - 7$ |

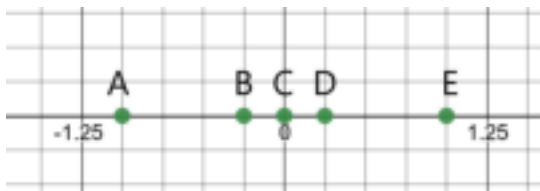
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| Chang Learning SAT Lesson 8: Graphing Linear & Quadratic Functions |
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SAT Lesson #8: Classwork (Continued):

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|--|--|---|---|----|---|------|----|---|---|----|---|
| 6) What is the intersection point of $y = 3(x - 2)^2 + 4$ and $y = 6x - 11$? | A) (1, 7) B) (2, 4) C) (3, 7) D) (4, 10) | | | | | | | | | | |
| 7) Which parabola is represented by the table shown ? <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;">4</td> </tr> <tr> <td style="padding: 5px;">f(x)</td> <td style="padding: 5px;">10</td> <td style="padding: 5px;">7</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">-5</td> </tr> </table> | x | 1 | 2 | 3 | 4 | f(x) | 10 | 7 | 2 | -5 | A) $f(x) = -x^2 + 9$ B) $f(x) = -x^2 + 10$ C) $f(x) = -x^2 + 11$ D) $f(x) = -x^2 + 12$ |
| x | 1 | 2 | 3 | 4 | | | | | | | |
| f(x) | 10 | 7 | 2 | -5 | | | | | | | |
| 8) What is the point of intersection of $x = 0.2y^2 - 2$ and $x = -0.2y^2 + 2$ on the positive y-axis ? | A) (0, 3) B) (0, 3.1) C) (0, $\sqrt{10}$) D) (0, $\sqrt{8}$) | | | | | | | | | | |
| 9) How many intersections do the graphs $y = -x^2 + 12$ and $y = (x + 1)^2 + 15$ have ? | A) 0 B) 1 C) 2 D) Infinitely many | | | | | | | | | | |
| 10) What is the vertex and y-intercept of $y = \frac{9}{16}(x + 4)^2 - 7$? | A) (-4, +7) and +2 B) (-4, -7) and +2 C) (+4, +7) and -2 D) (+4, -7) and -2 | | | | | | | | | | |

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| Chang Learning SAT Lesson 8: Graphing Linear & Quadratic Functions |
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SAT Lesson #8: Classwork (Continued):

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| <p>11) Tick marks are equally spaced on the number line above. Which of the lettered points has a base coordinate equal to $(-\frac{1}{2})^2$?</p>  | <p>A) B B) C C) D D) E</p> |
| <p>12) In the xy-plane, the line with equation $y = 5x - 10$ crosses the x-axis at the point with coordinates (a,b) with b equal to zero. What is the value of a ?</p> | <p>A) -10 B) -2 C) 0 D) 2</p> |
| <p>13) On a number line, not drawn, ABC are placed in that order. $AC = 24$ and $AB = BC$. Point D (not shown) is on the line between A and B such that $AD = DB$. What does DC equal ?</p> | <p>A) 6 B) 12 C) 18 D) 20</p> |
| <p>14) The letter H is symmetric with respect to two different lines, the horizontal and vertical center lines of the letter. Which of the following letters is symmetric with respect to at least two different lines ?</p> | <p>A) K B) M C) X D) Y</p> |
| <p>15) Let the function f be defined by $f(x) = x^2 + 12$. If m is a positive number such that $f(2m) = 2f(m)$, what is the value of m ?</p> | <p>A) $\sqrt{6}$ B) $2\sqrt{6}$ C) $2\sqrt{3}$ D) 3</p> |

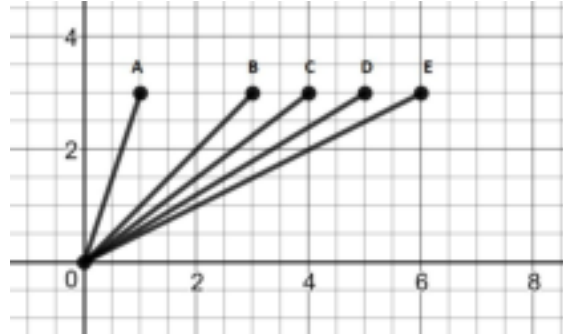
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SAT Lesson #8: Classwork (Continued):

16) Diane has a trip from her house to her job. Diana travels 16 miles from her home at $A(0,0)$ due east to $B(16,0)$, then 15 miles north from $B(16,0)$ to $C(16,15)$. Then 4 miles east again to point D . If she were able to travel from A to D directly, how much shorter, in miles, would the trip be ?

- A) 05 B) 10 C) 11 D) 15

17) In the figure below, what is the median of the slopes of OA , OB , OC , OD and OE ?



- A) $\frac{4}{3}$ B) 1 C) $\frac{3}{4}$ D) $\frac{3}{5}$

18) If triangle ABC has coordinates $A(2k, 2)$, $B(3.5k, 6)$ and $C(5k, 2)$ and has area 18, what is the value of k ?

- A) $\frac{3}{4}$ B) $\frac{6}{5}$ C) $\frac{3}{2}$ D) 3

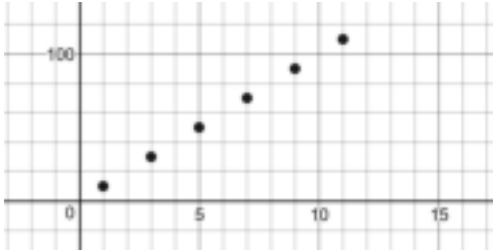
19) In the xy -coordinate system, $(p,0)$ is one of the points of intersection of the graphs of $y = x^2 - 9$. If p is positive, what is the value of p ?

- A) 3 B) 6 C) 9 D) 18

Chang Learning SAT Lesson 8: Graphing Linear & Quadratic Functions

SAT Lesson #8: Classwork (Continued):

20) A biology teacher graphed the length of a fish over time, and the results are shown above. If L represents the length of the fish in millimeters and W represents the number of the week, which of the following equations best describes the data shown ?



- A) $L = W$ B) $L = W + 10$
 C) $L = 10W$ D) $L = 10W + 10$

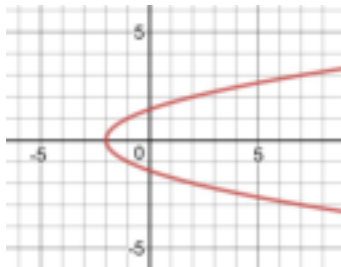
21) The quadratic function g is given by $g(x) = ax^2 + bx + c$, where a and c are negative constants. Which of the following attributes must be the graph of g ?

Grid In the sum of points for all true statements below:

| | |
|-----------|-----------------------------|
| 1 Point | Opens Up |
| 2 Points | Opens Down |
| 4 Points | Y Intercept is above origin |
| 8 Points | Y intercept is below origin |
| 16 Points | 2 real, rational roots |
| 32 Point | Two complex roots |

Grid In

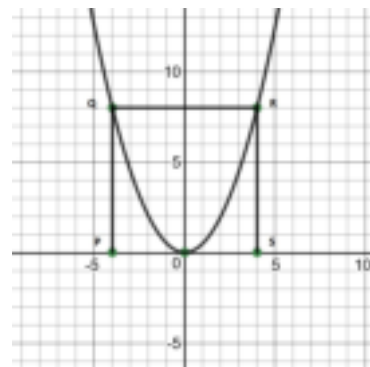
22) The graph shown has vertex at $(-2,0)$ and is symmetric about the x -axis. What is the y value for $x = 7$ that lies in the 4th quadrant ?



The equation for the parabola shown $x = y^2 - 2$

Grid In

23) In a parabola, PQRS is a square and point Q, R, and O lie on the graph of $y = ax^2$, where a is a constant. If the area of the square is 64, what is the value of a ?

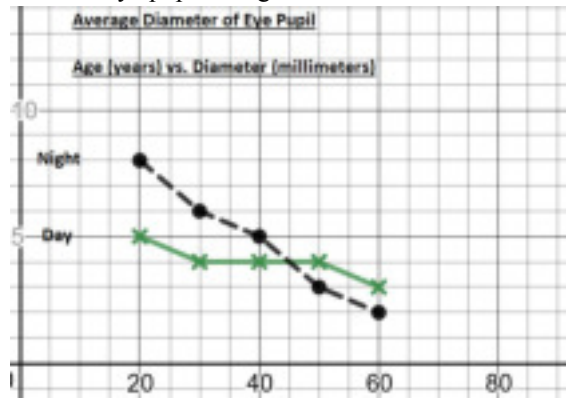


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| Chang Learning SAT Lesson 8: Graphing Linear & Quadratic Functions |
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SAT Lesson #8: Classwork (Continued):

- 24) The graph above shows the effect of aging on the diameter of the eye pupil. Based on the graph, what is the best estimate of the age at which the average diameter of the eye pupil during the day will equal the average diameter of the eye pupil at night ?



Grid In

- 25) If h and k are constant and $x^2 + kx + 7$ is equivalent to $(x + 1)(x + h)$, what is the value of k ?

Grid In

Chang Learning SAT Lesson 8: Graphing Linear & Quadratic Functions**SAT Lesson #8 Homework: Quadratic Formulas**

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| 1) Which of the following points is closest to the point $(x,y) = (-5, 7)$? | A) $(-10, -2)$ B) $(-3, 8)$ C) $(4, 7)$ D) $(-2, 9)$ |
| 2) What is the area of square ABCD with opposite corners at $A(5, 7)$ and $C(2, 4)$? | A) 9 B) 16 C) 25 D) 36 |
| 3) A lattice point has coordinates (x,y) that are both integers. How many lattice points are contained in $(x-3)^2 + (y-4)^2 \leq 6$, the circle centered at the origin with radius $\sqrt{6}$? | A) 15 B) 17 C) 19 D) 21 |
| 4) Which line is parallel to $y = \frac{-4}{3}x + 7$? | A) $y = \frac{-3}{4}x + 2$ B) $y = \frac{-4}{3}x - 5$ C) $y = \frac{3}{4}x + 1$ D) $y = \frac{4}{3}x + 7$ |
| 5) Which line is perpendicular to $y = 5x + 1$? | A) $y = \frac{1}{5}x + 4$ B) $y = \frac{1}{5}x + 7$ C) $y = -\frac{1}{5}x + 3$ D) $y = \frac{1}{5}x - 5$ |

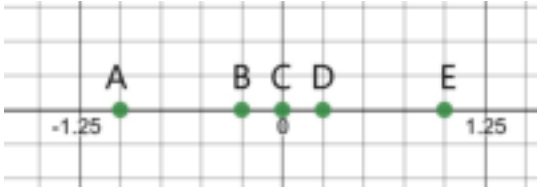
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Homework Problem Set (Continued):

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|--|---|----|----|-----|---|------|---|----|----|-----|--|
| 6) Which point is an intersection point of $y = -2(x - 4)^2 + 6$ and $y = 3x - 11$? | A) (0, -15) B) (1, -8) C) (2, 5) D) (5, 4) | | | | | | | | | | |
| 7) Which parabola is represented by the table shown ? <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;">4</td> </tr> <tr> <td style="padding: 5px;">f(x)</td> <td style="padding: 5px;">6</td> <td style="padding: 5px;">25</td> <td style="padding: 5px;">58</td> <td style="padding: 5px;">105</td> </tr> </table> | x | 1 | 2 | 3 | 4 | f(x) | 6 | 25 | 58 | 105 | A) $f(x) = 2x^2 + 19x - 15$ B) $f(x) = 3x^2 + 3$ C) $f(x) = 5x^2 + 1$ D) $f(x) = 7x^2 - 2x + 1$ |
| x | 1 | 2 | 3 | 4 | | | | | | | |
| f(x) | 6 | 25 | 58 | 105 | | | | | | | |
| 8) What is the point of intersection of $x = 0.3y^2 - 45$ and $x = -0.3y^2 + 45$ on the positive y-axis ? | A) (0, $5\sqrt{6}$) B) (0, 5) C) (0, 25) D) (0, $2\sqrt{30}$) | | | | | | | | | | |
| 9) How many intersections do the graphs $y = x^2 + 10x + 19$ and $y = \frac{1}{2}x - \frac{9}{2}$ have ? | A) 0 B) 1 C) 2 D) Infinitely many | | | | | | | | | | |
| 10) What is the vertex and y-intercept of $y = \frac{8}{9}(x + 3)^2 - 3$? | A) (3, -3) and +5 B) (3, 3) and -5 C) (-3, -3) and +5 D) (-3, -3) and -5 | | | | | | | | | | |

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| Chang Learning SAT Lesson 8: Graphing Linear & Quadratic Functions |
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SAT Lesson #8: Homework (Continued):

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| <p>11) Tick marks are equally spaced on the number line above. Which of the lettered points has a base coordinate equal to $(-\frac{1}{2})^2$?</p>  | <p>A) A B) C C) D D) E</p> |
| <p>12) In the xy-plane, the line with equation $y = 5x + 10$ crosses the x-axis at the point with coordinates (a,b). What is the value of a ?</p> | <p>A) -10 B) -2 C) 0 D) 2</p> |
| <p>13) On a number line, not drawn, ABC are placed in that order. $AC = 24$ and $AB = BC$. Point D (not shown) is on the line between A and B such that $AD = DB$. What does DC equal ?</p> | <p>A) 6 B) 12 C) 16 D) 18</p> |
| <p>14) The letter H is symmetric with respect to two different lines, the horizontal and vertical center lines of the letter. Which of the following letters is symmetric with respect to at least two different lines ?</p> | <p>A) K B) L C) N D) O</p> |
| <p>15) Let the function f be defined by $f(x) = x^2 + 18$. If m is a positive number such that $f(2m) = 2f(m)$, what is the value of m ?</p> | <p>A) 1 B) 2 C) 3 D) 4</p> |

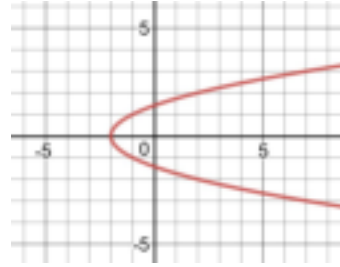
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SAT Lesson #8: Homework (Continued):

16) The quadratic function g is given by $g(x) = ax^2 + bx + c$, where a and c are positive constants. Which of the following could be the graph of g ?

- A) Opens up, vertex on the y -axis below the origin.
- B) Opens up, vertex in Quadrant II and y -intercept at the origin.
- C) Opens up, vertex in Quadrant IV and y -intercept above the origin.
- D) Opens up, vertex in Quadrant III and y -intercept below the origin.

17) The graph shown has vertex at $(-2,0)$ and is symmetric about the x -axis. Which of the following could be the equation of the parabola ?



- A) $x = 1.1y^2 - 2$
- B) $x = -1.1y^2 - 2$
- C) $x = 1.1(y - 2)^2$
- D) $x = 1.1(y - \sqrt{2})^2$

18) If triangle ABC has coordinates $A(2k, 2)$, $B(3.5k, 6)$ and $C(5k, 2)$ and has area 18, what is the value of k ?

- A) $\frac{3}{10}$ B) $\frac{3}{4}$ C) $\frac{3}{2}$ E) 3

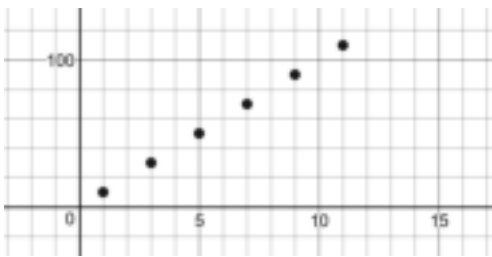
19) In the xy -coordinate system, $(p,0)$ is one of the points of intersection of the graph of $y = x^2 - 36$. If p is positive, what is the value of p ?

- A) 3 B) 6 C) 9 D) 18

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| Chang Learning SAT Lesson 8: Graphing Linear & Quadratic Functions |
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SAT Lesson #8: Homework (Continued):

- 20) A biology teacher graphed the length of a fish over time, and the results are shown above. If L represents the length of the fish in centimeters and W represents the number of the week, which of the following equations best describes the data shown?

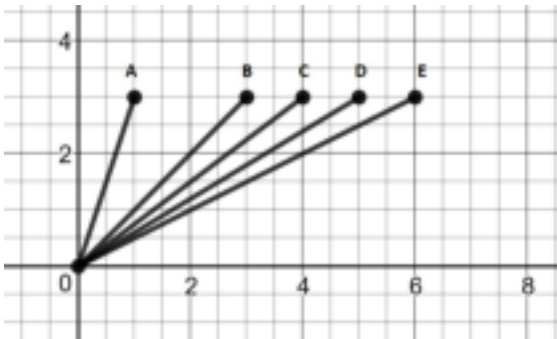


- A) $L = W$ B) $L = W + 10$
 C) $L = 10W$ D) $L = 10W + 10$

- 21) Diane has a trip from her house to her job. Diana travels 16 miles from her home at $A(0,0)$ due east to $B(16,0)$, then 15 miles north from $B(16,0)$ to $C(16,15)$. Then 4 miles east again to point D . If she were able to travel from A to D directly, how much shorter, in miles, would the trip be?

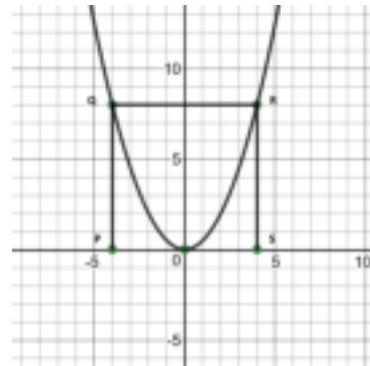
Grid In

- 22) In the figure below, what is the mean of the slopes of OA , OB , OC , OD and OE ?



Grid In

- 23) In a parabola, PQRS is a square and point Q , R , and O lie on the graph of $y = ax^2$, where a is a constant. If the perimeter of the square is 32, what is the value of a ?

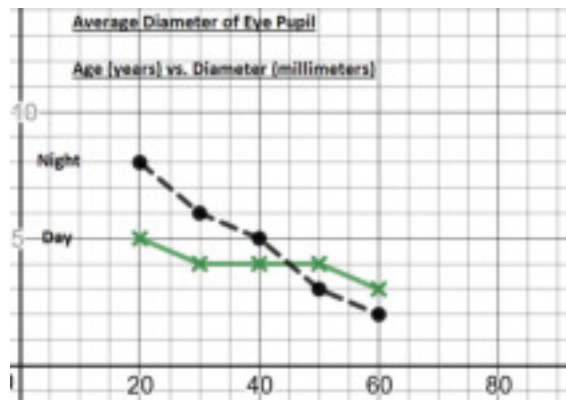


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SAT Lesson #8: Homework (Continued):

- 24) The graph above shows the effect of aging on the diameter of the eye pupil. Based on the graph, what is the best estimate of the age at which the average diameter of the eye pupil during the day will be 1 millimeter greater than the average diameter of the eye pupil at night ?



Grid In

- 25) If h and k are constant and $x^2 + kx + 6$ is equivalent to $(x - 1)(x + h)$, what is the value of k ?

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| Chang Learning SAT Lesson 8: Graphing Linear & Quadratic Functions |
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| Extra Credit Challenge Questions: |
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SAT Level 1

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| <p>1. Which of the following equations does NOT represent the line containing the points (15, 14) and (10, 10) ?</p> | <p>A. $y = \frac{4}{5}x + 2$ B. $y - 10 = \frac{4}{5}(x - 10)$ C. $y - 14 = \frac{4}{5}(x - 15)$ D. $4x - 5y + 10 = 0$ E. $4x + 5y = -10$</p> |
| <p>2. If $f(x) = \frac{x+3}{2}$ and f^{-1} is the inverse function of f, what is $f^{-1}(-5)$?</p> | <p>A. 1 B. 4 C. -13 D. 7 E. -7</p> |
| <p>3. In Mr. Taylor's first period geometry class. The mean score of 30 students on a test is 76 percent. In his second period class, the mean score of 22 students is 82 percent. What is the mean score of the 52 students ?</p> | <p>A. 77% B. 78% C. 78.5% D. 79% E. 79.5%</p> |
| <p>4. If 8 percent of an 18 gallon solution is chlorine, how many gallons of water must be added to make a new solution that is 6 percent chlorine ?</p> | <p>A. 6 gallons B. 8 gallons C. 10 gallons D. 12 gallons E. 18 gallons</p> |
| <p>5. $(2\sin(x))(9\sin(x)) - (6\cos(x))(-3\cos(x)) =$</p> | <p>A. 18 B. $18\sin^2(x) - 18\cos^2(x)$ C. $18\sin(x) + 18\cos(x)$ D. 36 E. 1</p> |

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| Chang Learning SAT Lesson 8: Graphing Linear & Quadratic Functions |
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| Extra Credit Challenge Questions: |
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SAT Level 2

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| 1. In the coordinate plane, if the distance between the point (x,y) and the origin is 3, what is the distance between the point $(2x,2y)$ and the origin ? | A. $\frac{3}{2}$ B. 3 C. 6 D. $6\sqrt{2}$ E. 9 |
| 2. For what value of k does the graph of the function f , given by $f(x) = \frac{3x+6}{x+k}$, NOT have a vertical asymptote ? | A. -3 B. -2 C. 0 D. 2 E. 3 |
| 3. For which of the following values of x in the interval $[0, 2\pi]$ is $f(x) = \frac{2}{2\sin x + 2}$ undefined ? | A. 0.73 B. 2.30 C. 2.41 D. 2.74 E. 3.87 |
| 4. For which of the following functions is $f(x+3) = f(x) * f(3)$ for all values of x in the domain of the function ? | A. $f(x) = x$ B. $f(x) = e^x$ C. $f(x) = \log x$ D. $f(x) = \sin x$ E. $f(x) = \tan x$ |
| 5. The sequence $\{a_n\}$ of complex numbers has the property that $\{a_n\} = i a_{n-1}$ for all n . If the 10th term of the sequence is $-2 + i$, what is the 99th term of the sequence ? | A. $+1 + 2i$ B. $-1 + 2i$ C. $-1 - 2i$ D. $+2 - i$ E. $-2 - i$ |