

Pythagorean Theorem

By now you have probably heard of the Pythagorean Theorem in your studies of mathematics. In 300 BCE, the mathematician Euclid wrote a 13 volume work on the subject that included the famous theorem. In this collection of geometric axioms and postulates, he lays the foundation for two and three dimensional geometry.

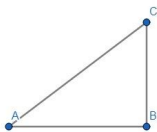
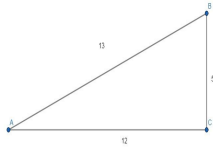

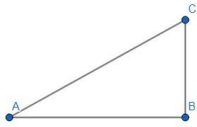
The Pythagorean Theorem:

In right angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.

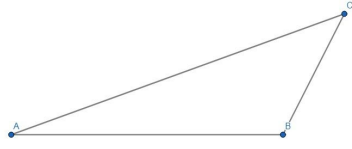
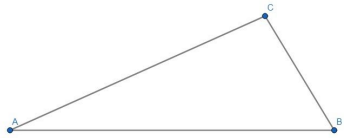
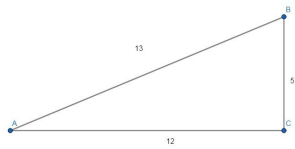
Euclid's Elements. Book I, Proposition 47. (300 BCE)

However, perhaps this important historic work was a collection of the times. Pre-dating this encyclopedia of knowledge was a babylonian clay tablet, Plimpton 322.

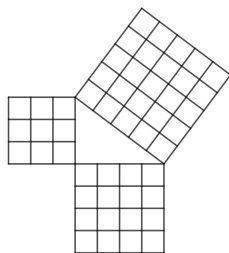
Example #1: What are some examples of Pythagorean triangles ?

<p>Three sides are $\{3, 4, 5\}$</p> $a^2 + b^2 = c^2$	<p>Three sides are $\{5, 12, 13\}$</p> $a^2 + b^2 = c^2$	<p>Three sides are $\{7, 24, 25\}$</p> $a^2 + b^2 = c^2$	<p>Three sides are $\{8, 15, 17\}$</p> $a^2 + b^2 = c^2$
			
$9 + 16 = 25$ $25 = 25$	$25 + 144 = 169$ $169 = 169$	$49 + 576 = 625$ $625 = 625$	$64 + 225 = 289$ $289 = 289$

Example #2: Which set of numbers could represent the side lengths of a right triangle ?

<p>Three sides are $\{5, 10, 13\}$</p> $a^2 + b^2 = c^2$	<p>Three sides are $\{5, 12, 10\}$</p> $a^2 + b^2 = c^2$	<p>Three sides are $\{5, 12, 13\}$</p> $a^2 + b^2 = c^2$
		
$25 + 100 \neq 169$ <p style="text-align: center;">NOT a right triangle</p>	$25 + 144 \neq 100$ <p style="text-align: center;">NOT a right triangle</p>	$25 + 144 = 169$ <p style="text-align: center;">A right triangle</p>

Example #3: Area Representation



Notice also that this “symbol of the Pythagorean school” also represents with squares and a right triangle that the area of the base legs squared equals the square of the hypotenuse (the side opposite the right angle).

$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

$$25 = 25$$



Example: Plimpton 322

Row #5:

$$65^2 + 72^2 = 97^2$$

$$4225 + 5184 = 9409$$

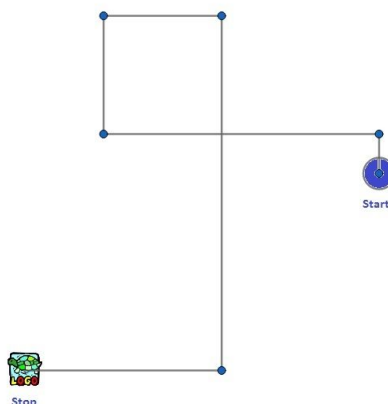
$$9409 = 9409$$

This clay tablet from the Babylonian Empire (300 BCE) contains a list of Pythagorean Triples in Sexagesimal (base 60) notation. The cuneiform wedge marks show a list that includes $\{3,4,5\}$ dilated $D_{15} = \{45,60,75\}$. Also $\{65,72,97\}$ and other integer values with decreasing slope ratios for each row representation. Was it for architectural use?

Example: Mapping a journey

Seymour programs his robot “Kame” to move with a computer program. He instructs Kame to move with the following directions:

Step	Direction
1	1 unit north
2	7 units west
3	3 units north
4	3 units east
5	9 units south
6	5 units west



The final displacement of the turtle is

9 units west, 5 units south.

Solution:

Make a right triangle with legs 9 and 5 units.

$$9^2 + 5^2 = d^2$$

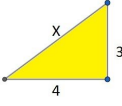
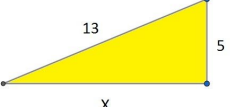
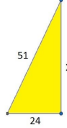
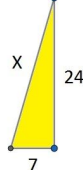
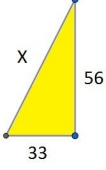
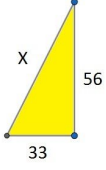
$$81 + 25 = d^2$$

$$106 = d^2$$

$$10.3 = d$$

The robot turtle is 10.3 units away from home.

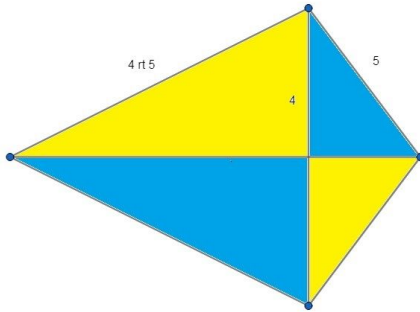
SHSAT Lesson #17 Classwork: Pythagorean Theorem

<p>1. What is the value of x, based on the Pythagorean theorem $a^2 + b^2 = c^2$?</p> <table border="1" data-bbox="204 445 665 508"> <tr> <td>$a = 4$</td> <td>$b = 3$</td> <td>$c = x$</td> </tr> </table>	$a = 4$	$b = 3$	$c = x$		<p>A. 4 B. 5 C. 6 D. 7 E. 8</p>
$a = 4$	$b = 3$	$c = x$			
<p>2. What is the value of x, based on the Pythagorean theorem $a^2 + b^2 = c^2$?</p> <table border="1" data-bbox="204 676 665 739"> <tr> <td>$a = x$</td> <td>$b = 5$</td> <td>$c = 13$</td> </tr> </table>	$a = x$	$b = 5$	$c = 13$		<p>A. 6 B. 8 C. 10 D. 12 E. 14</p>
$a = x$	$b = 5$	$c = 13$			
<p>3. What is the value of x, based on the Pythagorean theorem $a^2 + b^2 = c^2$?</p> <table border="1" data-bbox="204 907 665 970"> <tr> <td>$a = 24$</td> <td>$b = x$</td> <td>$c = 51$</td> </tr> </table>	$a = 24$	$b = x$	$c = 51$		<p>A. 32 B. 36 C. 40 D. 45 E. 50</p>
$a = 24$	$b = x$	$c = 51$			
<p>4. What is the value of x, based on the Pythagorean theorem $a^2 + b^2 = c^2$?</p> <table border="1" data-bbox="204 1129 665 1192"> <tr> <td>$a = 7$</td> <td>$b = 24$</td> <td>$c = x$</td> </tr> </table>	$a = 7$	$b = 24$	$c = x$		<p>A. 25 B. 26 C. 27 D. 28 E. 29</p>
$a = 7$	$b = 24$	$c = x$			
<p>5. What is the value of x, based on the Pythagorean theorem $a^2 + b^2 = c^2$?</p> <table border="1" data-bbox="204 1390 665 1453"> <tr> <td>$a = 33$</td> <td>$b = 56$</td> <td>$c = x$</td> </tr> </table>	$a = 33$	$b = 56$	$c = x$		<p>A. 43 B. 51 C. 58 D. 65 E. 97</p>
$a = 33$	$b = 56$	$c = x$			
<p>6. If each value in problem #5 is divided by 3, what is the new value of x, based on the Pythagorean theorem $a^2 + b^2 = c^2$?</p> <table border="1" data-bbox="204 1705 665 1768"> <tr> <td>$a = 33/3$</td> <td>$b = 56/3$</td> <td>$c = x$</td> </tr> </table>	$a = 33/3$	$b = 56/3$	$c = x$		<p>A. 43/3 B. 17 C. 58/3 D. 65/3 E. 65/9</p>
$a = 33/3$	$b = 56/3$	$c = x$			

SHSAT Lesson #17: Classwork (continued)

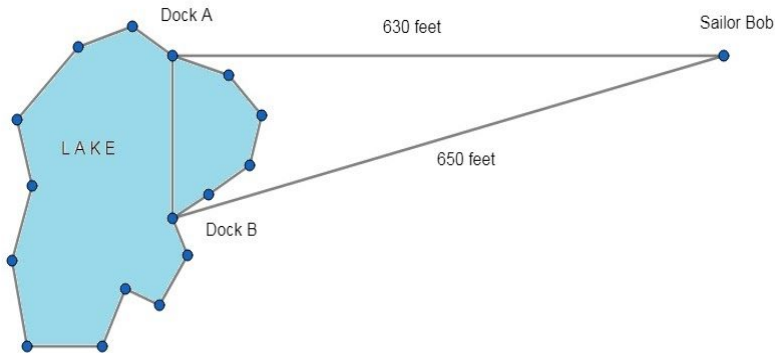
7. What is the hypotenuse of a right triangle with legs $a = 20$ and $b = 21$?	A. 22 B. 25 C. 28 D. 29 E. 31
8. What is the length of the hypotenuse of a right triangle with legs $a = 2$ and $b = \sqrt{21}$?	A. 4 B. 5 C. 6 D. 7 E. 8
9. A 15 foot ladder leans against a building with the bottom of the ladder anchored 4 feet from the base. How far up the side of the building does the ladder reach ? (Round your answer to the nearest tenth.)	A. 14.5 feet B. 14.2 feet C. 13.2 feet D. 12.6 feet E. 12.1 feet
10. A flagpole outside the school has a cable that a handler holds that is 37 feet long. The handler is standing 12 feet from the base of the pole. How tall is the flagpole ?	A. 31 B. 33 C. 35 D. 37 E. 39
11. Which numbers do <i>not</i> represent the sides of a right triangle ?	A. {6, 8, 10} B. {20, 21, 29} C. {11, 60, 61} D. {8, 14, 17} E. {33, 56, 65}
12. Which set of numbers could be the lengths of the sides of a right triangle ?	A. {3, 4, 6} B. {10, 24, 26} C. {15, 22, 28} D. {4, 8, 11} E. {5, 15, 17}

SHSAT Lesson #17: Classwork (continued)

<p>13. Which set of numbers could NOT represent the side lengths of a right triangle ?</p>	<p>A. $\{1, 3, \sqrt{10}\}$ B. $\{1, 4, \sqrt{17}\}$ C. $\{2, 5, \sqrt{29}\}$ D. $\{3, 7, \sqrt{58}\}$ E. $\{3, 8, \sqrt{77}\}$</p>
<p>14. Which set of numbers could NOT represent the side lengths of a right triangle ?</p>	<p>A. $\{\sqrt{3}, \sqrt{13}, 4\}$ B. $\{\sqrt{12}, \sqrt{13}, 5\}$ C. $\{\sqrt{8}, \sqrt{8}, 6\}$ D. $\{\sqrt{11}, \sqrt{38}, 7\}$ E. $\{\sqrt{13}, \sqrt{51}, 8\}$</p>
<p>15. A radio tower that is 25 feet tall has 3 cables that keep the tower stable. Each cable is attached 5 feet below the top and is anchored 15 feet from the base of the tower. How much cable is supporting the tower ?</p>	<p>A. 25 B. 50 C. 60 D. 75 E. 100</p>
<p>16. The kite below is designed with a cross brace that makes 4 right triangles. What is the length of the kite ? <i>measurements are $\{4, 5 \text{ and } 4\sqrt{5}\}$</i></p> <p style="text-align: center;"><<<----- Length of Kite ----->>></p> 	<p>A. 9 B. 10 C. 11 D. 12 E. 13</p>

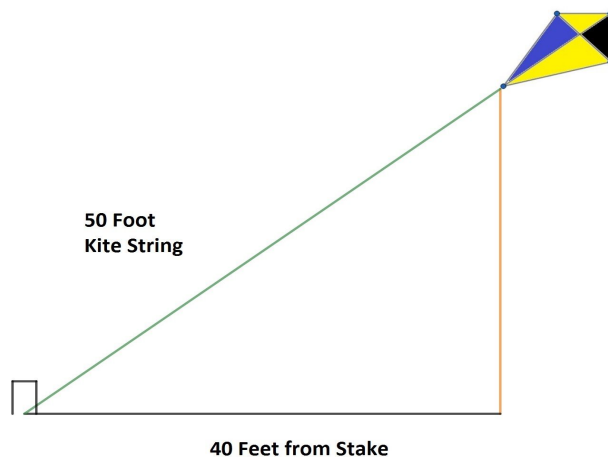
SHSAT Lesson #17: Classwork (continued)

17. Sailor Bob would like to know how far it is by Boat across a lake, so he walks away from the charted course at Dock A a right angle, then when he sees Dock B around the rocky coastline, he walks a line to Dock B. Since a right triangle was formed, what is the Boat distance AB across the lake ?



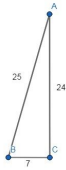
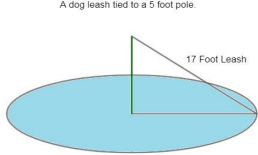
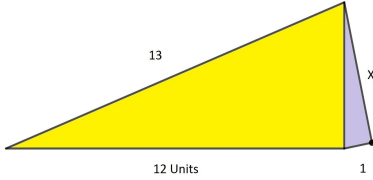
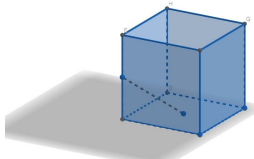
- A. 120 feet
 B. 150 feet
 C. 180 feet
 D. 210 feet
 E. None of these.

18. Mary and Betty-Ann decide to fly a kite together. Mary ties the kite to a stake they plant in a field, and her friend Betty-Ann walks 40 feet away from the stake and releases the kite, which rises over her head. It's very windy and the 50 foot string is under tension and straight as a bow string. How high is the kite above Betty-Ann ?



- A. 25 Feet
 B. 30 Feet
 C. 35 Feet
 D. 40 Feet
 E. None of these.

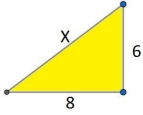
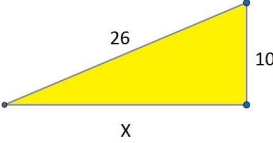
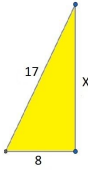
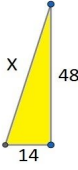
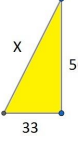
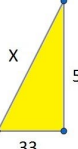
SHSAT Lesson #17: SHSAT Exam Classwork:

<p>19. Mary-Jo has to hurry home, so she decides to walk across a rectangular field that is 80 feet by 150 feet. How much shorter is the distance along the diagonal than walking along the edge of the field from one opposite corner to the other ?</p>	<div style="border: 1px solid black; padding: 10px; text-align: center;"> <p>Rectangular 80 ft by 150 ft field</p> </div>	<p>A. 60 ft B. 50 ft C. 40 ft D. 30 ft E. 20 ft</p>
<p>20. A fireman's ladder is 50 feet long. How far from the base of a tall building must the ladder be anchored so that the ladder will reach 48 feet from the bottom of the building ?</p>		<p>A. 30 ft B. 24 ft C. 20 ft D. 16 ft E. 14 ft</p>
<p>21. What is the hypotenuse of a right triangle with legs of length 6 and 12 ?</p>		<p>A. 16 ft B. $6\sqrt{5}$ ft C. $12\sqrt{3}$ ft D. $9\sqrt{2}$ ft E. 15 ft</p>
<p>22. A dog is tied to a 5 foot pole at the top, and has a 17 foot leash. What is the square footage of the area the dog can roam ?</p>	<p>A dog leash tied to a 5 foot pole.</p> 	<p>A. 264π B. 274π C. 294π D. 314π E. 324π</p>
<p>23. In the diagram at right, both triangles are right triangles. What is the value of x ?</p>		<p>A. $\sqrt{22}$ B. $\sqrt{24}$ C. $\sqrt{30}$ D. $\sqrt{32}$ E. $\sqrt{34}$</p>
<p>24. A cube of side length 5 is shown at right. What is the distance between the center of the base of the cube and the midpoint of the vertical edge, as shown ?</p>		<p>A. $\sqrt{5}$ B. $\frac{\sqrt{5}}{2}$ C. $\frac{5\sqrt{3}}{2}$ D. $\sqrt{6}$ E. $5\sqrt{2}$</p>

SHSAT Lesson #17: SHSAT Mixed Review Classwork:

<p>25. Ann's tree is 76 inches tall and is growing at a rate of 3.6 inches per year. Bob's tree is 58 inches tall and is growing at a rate of 4.8 inches per year. In how many years will Bob's tree reach Ann's tree in height ?</p>	Grid In
<p>26. Sam's bowling scores for seven games were, in order played, 54,35,89,76,48,x and y. Sam's mode score for the seven games was 76, and his median score was 54. If $48 < x < y$, state one possible value of x ?</p>	Grid In
<p>27. A sheet of rectangular paper measures 12 cm by 20 cm. At the edge of the paper, a margin exactly 1-cm wide is painted all the way around. What percentage of the area of the original rectangle remains unpainted ?</p>	Grid In
<p>28. If $\frac{4-x}{x-2} = \frac{-3}{x}$ then state one possible integer value for x ?</p>	Grid In
<p>29. In the x-y plane, the center of a circle is located at $(-8, -11)$. The point $(-8, -8.5)$ lies inside the circle, and the point $(-11, -7)$ lies outside the circle. If the circle's radius is an integer, what is its area ?</p> <p>(Solve in terms of pi. Grid final in your final answer as follows: $A = a\pi$. Example : $A = 30\pi$, grid in 30)</p>	Grid In
<p>30. Rectangle A measures 12 meters wide by x meters long and has an area equal to the area of Rectangle B, which measures 30 meters wide by 48 meters long. What is the perimeter of Rectangle A ?</p>	Grid In

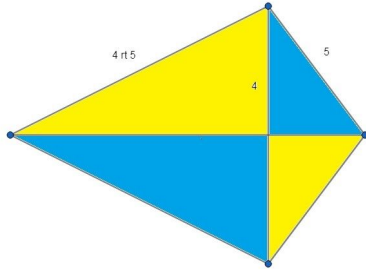
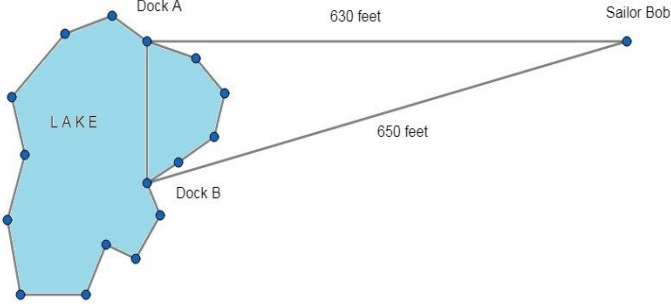
SHSAT Lesson #17 Homework: Pythagorean Theorem

<p>1. What is the value of x, based on the Pythagorean theorem $a^2 + b^2 = c^2$?</p> <table border="1" data-bbox="204 445 665 508"> <tr> <td>$a = 8$</td> <td>$b = 6$</td> <td>$c = x$</td> </tr> </table>	$a = 8$	$b = 6$	$c = x$		<p>A. 8 B. 10 C. 12 D. 14 E. 16</p>
$a = 8$	$b = 6$	$c = x$			
<p>2. What is the value of x, based on the Pythagorean theorem $a^2 + b^2 = c^2$?</p> <table border="1" data-bbox="204 672 665 735"> <tr> <td>$a = x$</td> <td>$b = 10$</td> <td>$c = 26$</td> </tr> </table>	$a = x$	$b = 10$	$c = 26$		<p>A. 20 B. 22 C. 24 D. 26 E. 28</p>
$a = x$	$b = 10$	$c = 26$			
<p>3. What is the value of x, based on the Pythagorean theorem $a^2 + b^2 = c^2$?</p> <table border="1" data-bbox="204 932 665 995"> <tr> <td>$a = 8$</td> <td>$b = x$</td> <td>$c = 17$</td> </tr> </table>	$a = 8$	$b = x$	$c = 17$		<p>A. 11 B. 13 C. 15 D. 17 E. 19</p>
$a = 8$	$b = x$	$c = 17$			
<p>4. What is the value of x, based on the Pythagorean theorem $a^2 + b^2 = c^2$?</p> <table border="1" data-bbox="204 1167 665 1230"> <tr> <td>$a = 14$</td> <td>$b = 48$</td> <td>$c = x$</td> </tr> </table>	$a = 14$	$b = 48$	$c = x$		<p>A. 50 B. 52 C. 54 D. 56 E. 58</p>
$a = 14$	$b = 48$	$c = x$			
<p>5. What is the value of x, based on the Pythagorean theorem $a^2 + b^2 = c^2$?</p> <table border="1" data-bbox="204 1407 665 1470"> <tr> <td>$a = 33$</td> <td>$b = 56$</td> <td>$c = x$</td> </tr> </table>	$a = 33$	$b = 56$	$c = x$		<p>A. 43 B. 51 C. 58 D. 65 E. 97</p>
$a = 33$	$b = 56$	$c = x$			
<p>6. If each value in problem #5 is divided by 5, what is the new value of x, based on the Pythagorean theorem $a^2 + b^2 = c^2$?</p> <table border="1" data-bbox="204 1663 665 1726"> <tr> <td>$a = 33/5$</td> <td>$b = 56/5$</td> <td>$c = x$</td> </tr> </table>	$a = 33/5$	$b = 56/5$	$c = x$		<p>A. 19.6 B. 13 C. 14.5 D. 12.75 E. 10.75</p>
$a = 33/5$	$b = 56/5$	$c = x$			

SHSAT Lesson #17: Homework (continued)

<p>7. What is the hypotenuse of a right triangle with legs $a = 48$ and $b = 55$?</p>	<p>A. 65 B. 73 C. 84 D. 85 E. 97</p>
<p>8. What is the length of the hypotenuse of a right triangle with legs $a = 9$ and $b = \sqrt{19}$?</p>	<p>A. 6 B. 8 C. 10 D. 12 E. 14</p>
<p>9. A 18 foot ladder leans against a building with the bottom of the ladder anchored 6.5 feet from the base. How far up the side of the building does the ladder reach ? (round to the nearest hundredth)</p>	<p>A. 16.31 feet B. 16.43 feet C. 16.58 feet D. 16.79 feet E. 16.96 feet</p>
<p>10. A flagpole in central park stands tall at 45 feet. A park employee holds a rope from the flag atop that is held straight in tension and is 53 feet long. How far is the person standing from the base of the flagpole ?</p>	<p>A. 14 feet B. 15 feet C. 18 feet D. 20 feet E. 28 feet</p>
<p>11. Which numbers do <i>not</i> represent the sides of a right triangle ?</p>	<p>A. {56, 90, 106} B. {1771, 2700, 3229} C. {161, 240, 289} D. {1679, 2400, 2929} E. {45,64,75}</p>
<p>12. Which set of numbers could be the lengths of the sides of a right triangle ?</p>	<p>A. {1, 3, 3} B. {12, 16, 20} C. {18, 80, 81} D. {7, 24, 26} E. {8, 13, 17}</p>

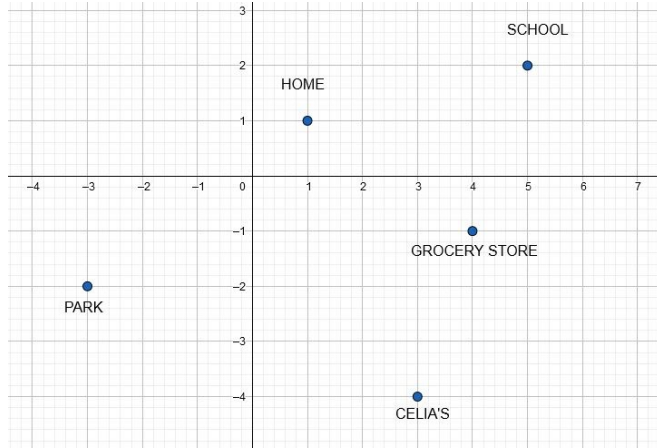
SHSAT Lesson #17: Homework (continued)

<p>13. Which set of numbers could NOT represent the side lengths of a right triangle ?</p>	<p>A. $\{\sqrt{10}, \sqrt{61}, 9\}$ B. $\{\sqrt{65}, \sqrt{35}, 10\}$ C. $\{102, \sqrt{19}, 11\}$ D. $\{\sqrt{110}, \sqrt{34}, 12\}$ E. $\{\sqrt{110}, \sqrt{59}, 13\}$ F.</p>
<p>14. A radio tower that is 25 feet tall has 3 cables that keep the tower stable. Each cable is attached 10 feet below the top and is anchored 20 feet from the base of the tower. How much cable is supporting the tower ?</p>	<p>A. 50 B. 60 C. 75 D. 90 E. 105 F.</p>
<p>15. The kite below is designed with a cross brace that makes 4 right triangles. What is the area of the kite ? (The 2 large right $\{x, 4, 4\sqrt{5}\}$ and 2 small right triangles $\{y, 4, 5\}$ are congruent.)</p> 	<p>A. 44 B. 48 C. 52 D. 56 E. 60</p>
<p>16. Sailor Bob would like to know how far it is by Boat across a lake, so he walks away from the charted course at Dock A a right angle, then when he sees Dock B around the rocky coastline, he walks a line to Dock B. He buys a commuter book with 20 one way tickets. What is the total distance of all the boat trips ?</p> 	<p>A. 3200 feet B. 3000 feet C. 2400 feet D. 2100 feet E. 1920 feet</p>

SHSAT Lesson #17: Homework (continued)

17. Below is a map of a local neighborhood. You can only walk on sidewalks along the straight lines shown. A student starts at home and goes to school, then after school walks with a friend, Celia with others to a park. Then they all go to a grocery store to shop. Then the friends all say goodbye at the store, and the student walks home. If the student took the direct route to each location, how many blocks total did the student walk in these commutes ?

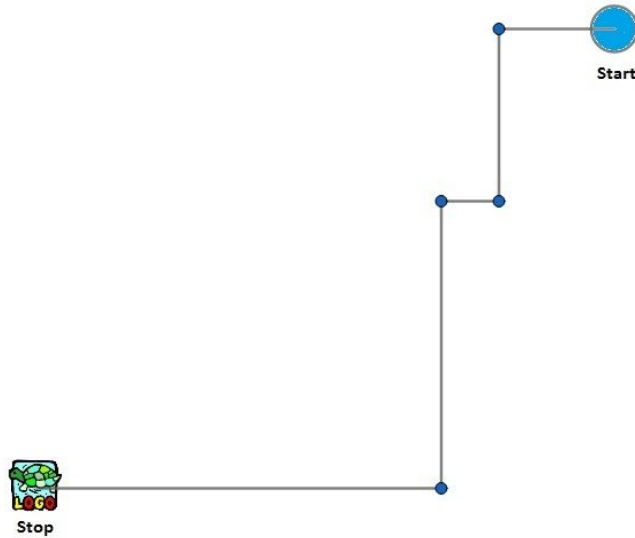
- A. 20 blocks
- B. 25 blocks
- C. 30 blocks
- D. 40 blocks
- E. 60 blocks




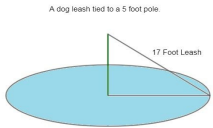
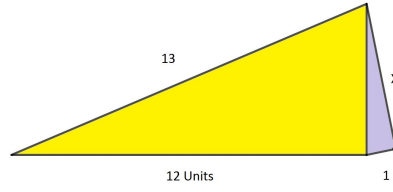
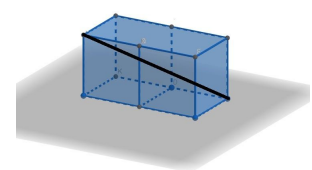
18. The robot turtle “Kame” is programmed to deliver a present along a route. How far away is Kame the Turtle from home ?

- A. 12.8 feet
- B. 13.4 feet
- C. 13.9 feet
- D. 14.6 feet
- E. 15.3 feet

Step	1	2	3	4	5
Command	Left 2	Down 3	Left 1	Down 5	Left 7



SHSAT Lesson #17: SHSAT Exam Homework:

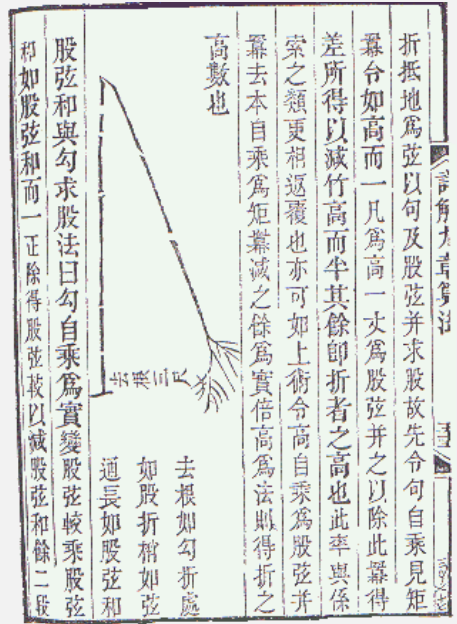
<p>19. Mary-Jo has to hurry home, so she decides to walk across a rectangular field that is 80 feet by 150 feet. How much longer is the distance along the edge of the field from one opposite corner to the other, than walking across the field diagonal ?</p>	<div style="border: 1px solid black; padding: 10px; text-align: center;"> <p>Rectangular 80 ft by 150 ft field</p> </div>	<p>A. 60 ft B. 50 ft C. 40 ft D. 30 ft E. 20 ft</p>
<p>20. A fireman's ladder is 25 feet long. How far from the base of a tall building must the ladder be anchored so that the ladder will reach 24 feet from the bottom of the building ?</p>		<p>A. 15 ft B. 12 ft C. 10 ft D. 8 ft E. 7 ft</p>
<p>21. What is the hypotenuse of a right triangle with legs of length 12 and 6 ?</p>		<p>A. 16 ft B. $6\sqrt{5} \text{ ft}$ C. $12\sqrt{3} \text{ ft}$ D. $9\sqrt{2} \text{ ft}$ E. 15 ft</p>
<p>22. A dog is tied to a 5 foot pole at the top, and has a 17 foot leash. What is the square footage of the area the dog can roam ?</p>		<p>A. 264π B. 274π C. 294π D. 314π E. 324π</p>
<p>23. In the diagram at right, both triangles are right triangles. What is the total area of the figure ? (Left triangle has base 12 and hypotenuse 13. Right triangle has base 1 and height x.)</p>		<p>A. 64.90 B. 59.15 C. 48.24 D. 32.45 E. None of these</p>
<p>24. A double cube is constructed by placing adjacent cubes of side length 5 as shown at right. What is the distance of the long diagonal, as shown ?</p>		<p>A) $6\sqrt{5}$ B) $\frac{10\sqrt{5}}{2}$ C) $\frac{5\sqrt{3}}{2}$ D) $5\sqrt{6}$ E) $10\sqrt{2}$</p>

SHSAT Lesson #17: SHSAT Mixed Review Classwork:

<p>25. At 12:00 noon a pool is $\frac{3}{25}$ full. It is being filled at a constant rate of 2 gallons per minute. At 2:30 pm the pool is 28% full. How many gallons does the pool hold ?</p>	Grid In
<p>26. There are at least three times as many boys in class as girls in a class. On a test, the boy's average score is 78, and the girl's average score is 94. Which of the following could be the average score for the entire class ?</p>	Grid In
<p>27. What is the ratio of $\frac{3}{5}$ to $\frac{6}{11}$?</p> <p>(Grid in your final answer as "a / b")</p>	Grid In
<p>28. If $5b=6c=7d=120$, then what is the value of $(7bd)$ divided by c ?</p>	Grid In
<p>29. Five people in a room weigh an average of 120 pounds each. When a sixth person enters, the average weight in the room increases to 125 pounds per person. What does the 6th person weigh ?</p>	Grid In

Question #30:

Homework Question #30: This problem is from a Chinese mathematics textbook called Chui-chang suan-shu, or Nine Chapters on the Mathematical Art, which was written about 250 B.C. A 10-ft-long stem of bamboo is broken in such a way that its tip touches the ground $t = 3$ ft from the base of the stem, as shown in the figure. What is the height of the break? (Hint: Use the Pythagorean Theorem.)

 <p style="text-align: center;">折抵地爲弦以句及股弦并求股故先令句自乘見矩 羅合如高而一凡爲高一丈爲股弦并之以除此羅得 差所得以減竹高而半其餘卽折者之高也此率與係 索之類更相遞覆也亦可如上術令高自乘爲股弦并 羅去本自乘爲矩羅減之餘爲實倍高爲法則得折之 高數也</p> <p style="text-align: center;">去根如勾折處 如股折梢如弦 通長如股弦和</p> <p style="text-align: center;">股弦和與勾求股法曰勾自乘爲實變股弦較乘股弦 和如股弦和一正除得股較以減股弦和餘二股</p>	<p>Grid In</p>
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Extra Credit Challenge Questions:

- Three numbers a , b , c , none zero, form an arithmetic progression. Increasing a by 1 or increasing c by 2 results in a geometric progression. Then b equals:
(A) 16 (B) 14 (C) 12 (D) 10 (E) 8
- If $x_{k+1} = x_k + \frac{1}{2}$ for $k = 1, 2, \dots, n-1$ and $x_1 = 1$, find $x_1 + x_2 + \dots + x_n$.
(A) $\frac{n+1}{2}$ (B) $\frac{n+3}{2}$ (C) $\frac{n^2-1}{2}$ (D) $\frac{n^2+n}{4}$ (E) $\frac{n^2+3n}{4}$
- Each side of triangle ABC is 12 units. D is the foot of the perpendicular dropped from A on BC , and E is the midpoint of AD . The length of BE , in the same unit, is:
(A) $\sqrt{18}$ (B) $\sqrt{28}$ (C) 6 (D) $\sqrt{63}$ (E) $\sqrt{98}$
- Any five points are taken inside of a square of side 1. Let a be the *smallest* possible number with the property that it is always possible to select one pair of points from these five such that the distance between them is equal to or less than a . Then a is :
(A) $\sqrt{3}/3$ (B) $\sqrt{2}/2$ (C) $2\sqrt{2}/3$ (D) 1 (E) $\sqrt{2}$
- If $3x^3 - 9x^2 + kx - 12$ is divisible by $x - 3$, then it is also divisible by
(A) $3x^2 - x + 4$ (B) $3x^2 - 4$ (C) $3x^2 + 4$ (D) $3x - 4$ (E) $3x + 4$