Name: **Date:**

Chang Learning **SHSAT Lesson #1: Number Sense Part 1**

Number Sense Part 1

This course is designed to help you prepare for the Specialized High School Achievement Test, which is given annually in the Fall Semester to NYC students. This is a merit based test, and each of the schools has a qualifying score. If you answer enough problems correctly, you will receive a letter of acceptance to attend one of the specialized High Schools in the five boroughs.

This test includes problems that are often not taught within the traditional middle school setting. It is good to be exposed to these problem solving questions in mathematics that are a large part of the SHSAT test. These are questions that often target several mathematics skills from your learning and bridge several units from your education. You have to think beyond the classroom.

Welcome to SHSAT Preparation Class!

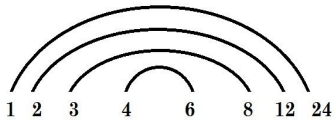
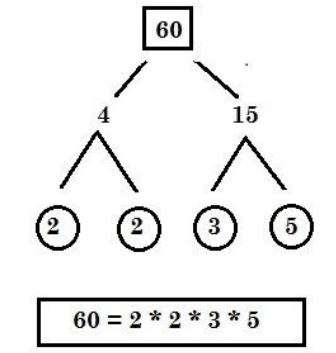
Number Sets

Natural Numbers	Counting numbers such as 1,2,3,4, or 5 apples. 1,2 or 3 trees.	$\{1, 2, 3, 4, 5 \ldots\}$
Whole Numbers	Counting numbers including zero.	$\{0, 1, 2, 3, 4, 5 \ldots\}$
Integers	Whole numbers and their opposites.	$\{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$
Rational Numbers	A ratio of two quantities can be written as *"a:b"*, *"a is to b"*, or in modern notation "$\frac{a}{b}$". This set includes percentages, terminating decimals and mixed numbers.	Examples: Modern Notations $\{-5.1, \ -2\frac{1}{3}, \ 0, \ 1.\overline{9}, \ 350\%, \ 10\}$
Irrationals: Square Roots	This is the side length of a SQUARE with *Area = A units squared.* Those areas that are not perfect squares, or ratios of perfect squares, are irrational. Examples: $\sqrt{1} = 1$, $\sqrt{4} = 2$ *but* $\sqrt{3}$ is irrational. The decimal pattern does NOT repeat. The golden ratio is an irrational number.	Examples: $\sqrt{1.5}, \sqrt{2}, \frac{1+\sqrt{5}}{2}, \sqrt{3}, \sqrt{\frac{9}{2}}, \sqrt{101}$
Irrationals: Transcendentals	Numbers on the number line that are NOT "roots of a polynomial function". $ln(x)$ = *The natural logarithm* $\Gamma(n)$ = *The gamma function, an interpolation of n!* e = *The constant called Euler's number* π = *The ratio constant "pi", pronounced "pie"*	Examples: $ln(2)$ 0.69314... $\Gamma(\frac{5}{2})$. 1.32934... e 2.71828... π 3.14159...

Name: **Date:**

Chang Learning	**SHSAT Lesson #1: Number Sense Part 1**

Special Number Subsets

Positives and Negatives	The positive integers and their opposites. There are a few rules that you should be familiar with: Multiplying by negative 1, absolute value and the additive inverse.	$7 * (-1) = -7$ $\|19\| = 19. \ \|-19\| = 19.$ $13 + (-13) = 0$
Even & Odd	Even numbers can be written in the form $2n$, where n is an integer. Odd numbers are commonly written as $2n+1$.	**Even Number Set** $\{\ldots -4, -2, 0, 2, 4, 6, 8\ldots.\}$ **Odd Number Set** $\{\ldots -3, -1, 1, 3, 5, 7, 9\ldots\}$
Factors	A list of the natural numbers that divide a number n without a remainder.	**Factors of 24** 
Primes	A prime number has factors that are only 1 and itself. 2 is the only even prime.	**Prime Factorization of 60** 
Consecutive Integers	Numbers that follow one another. A basic example is "pages in a book". There are also even and odd consecutives. A multiplication table is another example of a consecutive sequence of integers.	**Consecutive** $\{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4\ldots\}$ **Consecutive Even** $\{\ldots -4, -2, 0, 2, 4, 6, 8\ldots.\}$ **Consecutive Odd** $\{\ldots -3, -1, 1, 3, 5, 7, 9\ldots\}$
Multiples	A multiplication table is a list of multiples. The Least Common Multiple LCM is useful in comparing when two or more rates have common values. A basic example is working with two gears, one with 8 teeth and the other set with 14 teeth. The gears will "align" again after 56 gear increments.	**LCM {8,14}** See table below The LCM is 56.

	1	2	3	4	5	6	7
8	8	16	24	32	40	48	56
14	14	28	42	56			

Name: **Date:**

Chang Learning **SHSAT Lesson #1: Number Sense Part 1**

Divisibility Rules

If you've forgotten - or never learned - divisibility rules, spend a little time with this chart. Even if you know the rules, take a moment to review. Remember, there are no easy divisibility rules for the number 7.

Divisible By	The Rule	Example: 63,558
2	*The last digit is even.*	A multiple of 2 because 8 is even.
3	*The sum of the digits is a multiple of 3.*	A multiple of 3 because 6+3+5+5+8 = 27 which is a multiple of 3. .
4	*The sum of the digits comprise a 2-digit multiple of 4.*	NOT a multiple of 4 because 58 is not a multiple of 4 .
5	*The last digit is 5 or 0.*	NOT a multiple of 5 because it doesn't end in 5 or 0. .
6	*The last digit is even AND the sum of the digits is a multiple of 3.*	A multiple of 6 because it's a multiple of both 2 and 3.
8	*The last 3 digits are divisible by 8.*	NOT a multiple of 8, as $558 \div 8 = 69r4$.
9	*The sum of the digits is a multiple of 9.*	A multiple of 9 because 6+3+5+5+8 = 27 which is a multiple of 9.
10	*The last digit is 0.*	Not a multiple of 10 because it doesn't end in 0.
11	*Starting on the right, alternately subtract & add the digits. The final result is a multiple of 11 or 0.*	A multiple of 11 because $8-5+5-3+6=11$.

Order of Operations (PEMDAS)

The algebraic steps on the SHSAT follow the "order of operations" in mathematics. Problems are solved using the following order: Parenthesis, exponents, multiplication & division, then addition & subtraction.

P lease
E xcuse
M y
D ear
A unt
S ally

Or

P andas
E at
M ustard
D umplings **&**
A pple
S auce

Examples:

$2(15-2) \div 4 + 3^2 * (4 \div 12) - 1.5 =$
$2(13) \div 4 + 3^2 * (1/3) - 1.5 =$
$2(13) \div 4 + 9 * (1/3) - 1.5 =$
$26 \div 4 + 3 - 1.5 =$
$6.5 + 3 - 1.5 =$
$9.5 - 1.5 =$
8.

$[35(2^3 - 6) - 7(1 + 2 + 3)] \div 2 =$
$[35(8 - 6) - 7(1 + 2 + 3)] \div 2 =$
$[35(8 - 6) - 7(6)] \div 2 =$
$[35(2) - 7(6)] \div 2 =$
$[70 - 42] \div 2 =$
$[28] \div 2 =$
14.

Examples:

Parenthesis
Exponents
Multiply
Divide
Add
Subtract

Parenthesis:
- **Exponent**
- **Add & Subtract**
- **Multiply**
- **Subtract**

Divide

SHSAT Lesson #1 Classwork: Number Sense Part 1

1. $\frac{4}{5} + \frac{1}{7} - \frac{1}{2} =$	A. $\frac{31}{70}$ B. $\frac{3}{10}$ C. $\frac{1}{2}$ D. $\frac{33}{70}$ E. $\frac{41}{70}$
2. The number 84 is divisible by	A. 2,3,4,9 B. 2,3,4,5,6 C. 2,3,4,6,7 D. 2,3,4,5,6,7,8 E. 2,3,4,5,6,7,8,9
3. What is the value of $\lvert x - y \rvert + \lvert y + x \rvert$ *if* $x = (-5)$ *and* $y = (2)$?	A. 10 B. 14 C. 0 D. 6 E. -10
4. $100 + 5^2 + 5^2 + 5^2$	A. 5^3 B. $5^3 + 3(5^2)$ C. $10^2 + 5^6$ D. $7(5^2)$ E. (5^7)
5. If $x = 9$, what is the value of $\frac{3(7+x)}{(15-x)}$?	A. 10 B. 8 C. 6 D. 4 E. 2
6. Which of the following could be the sum of exactly three consecutive integers? Note: Consecutive integers are "Three in a row" such as {4,5,6} or {9,10,11}	A. 119 B. 121 C. 122 D. 123 E. 124

SHSAT Lesson #1: Classwork (continued)

7. X: $\frac{2}{3} > \frac{3}{5}$, Y: $9.01 < \sqrt{99}$, Z: $\sqrt{64} < 8.15$	A. X,Y and Z are all true B. Z is true C. X,Y and Z are all false D. Y is false E. X and Z are false
8. If p is an odd number, which of the following expressions MUST also be an odd number ?	A. $3p + 5$ B. $3p + 19$ C. $4p + 2$ D. $2(p + 6)$ E. $3(p + 4)$
9. A local deli sells pastrami for \$12.99 per pound. How much does 2.5 pounds of pastrami cost ? (Rounded to the nearest cent)	A. \$19.49 B. \$25.98 C. \$28.59 D. \$32.48 E. \$38.97
10. What is the Least Common Multiple (LCM) of 2,3 and 5?	A. 15 B. 24 C. 25 D. 30 E. 60
11. How many prime numbers are less than 30 ?	A. 14 B. 13 C. 12 D. 11 E. 10
12. A rectangle has a width that is twice the length. What is the perimeter of the rectangle is the width is 7 centimeters ?	A. 10.5 B. 21 C. 35 D. 42 E. 49

SHSAT Lesson #1: Classwork (continued)

13. The product of 5 numbers is positive. At most, how many of these 5 numbers are negative ?	A. 5 B. 4 C. 3 D. 2 E. 1
14. What is the smallest integer that can be added to 6,598 to make the result divisible by 9 ?	A. 0 B. 3 C. 8 D. 7 E. 2
15. Two infinite sets M & N are given. Which number belongs to BOTH sets ? M={the set of prime numbers} N={number whose digits add up to 5}	A. 32 B. 50 C. 41 D. 83 E. 31
16. Simplify $\lvert 2(5)^2 - (35 \div 7) \rvert =$	A. 20 B. 22 C. 95 D. 43 E. 45
17. The symbol *"factorial of n"* is written *n!* For example : 3! = 3*2*1= 6 4! = 4*3*2*1 = 24 What is the value of 3 times (5!) ?	A. 72 B. 120 C. 240 D. 360 E. 720
18. If M is an even positive integer and N is an odd positive integer, which of the following values must be odd ?	A. $M^2 - N^2 + 5$ B. $M^2 + N + 1$ C. $2(M - N)$ D. $M^2 - N^2$ E. $M^2 + 2N^2$

SHSAT Lesson #1: SHSAT Exam Classwork:

19. $\frac{1}{2} + \frac{2}{3} - \frac{3}{4} =$	A. $\frac{1}{3}$ B. $\frac{7}{12}$ C. $\frac{5}{12}$ D. 1 E. $\frac{11}{12}$
20. The product of the first 5 primes must be divisible by	A. 12 B. 18 C. 44 D. 60 E. 15
21. $\overline{.6} * \overline{.6} =$	A. .36 B. $\overline{.36}$ C. $\overline{.45}$ D. $\overline{.4}$ E. $1.\overline{2}$
22. $(16 - 3^2)^2 =$	A. 169 B. 100 C. 81 D. 64 E. 49
23. Find the smallest positive integer that is a multiple of both 14 and 30.	A. 420 B. 210 C. 120 D. 105 E. 60
24. Which of the following numbers is *less than* $\frac{1}{3}$?	A. $\frac{111}{333}$ B. $\frac{222}{666}$ C. $.1 * 4$ D. $.3 * 1$ E. $\frac{5!}{3!2!}$

SHSAT Classwork: Grid In Questions

25. How many prime numbers are multiplied together to make 60 ? Note: Prime numbers are divisible by 1 and themselves {2,3,5,7,11,13…} are the first few primes.	Grid In
26. Circle A is a set that contains all perfect square numbers; Circle B contains all factors of 100, and Circle C contains all composite numbers. How many numbers belong to all three circles ?	Grid In
27. If 40 consecutive multiples of 5 are written down in order, what could be the difference between the largest and the smallest number of the set ?	Grid In
28. Divide: $3\frac{1}{7} \div \frac{5}{7}$	Grid In
29. In the (x, y) coordinate plane, what is the distance between points $(-3, 10)$ and $(15, 10)$?	Grid In
30. Evaluate $(x+y)(x^2-xy+y^2)$ when $x = 10$ *and* $y = 1$	Grid In

SHSAT Lesson #1 Homework: Number Sense Part 1

1. If a and b are prime numbers, which of the following **cannot** be the sum of a and b? Note: Prime numbers are divisible by 1 and themselves only {2,3,5,7,11,13…} are the first few primes.	A. 15 B. 14 C. 13 D. 11 E. 9
2. Let letter ***H*** stand for a nonzero digit. The three-digit number ***HHH*** is always divisible by:	A. 2 B. 3 C. 5 D. 11 E. 13
3. How many integers are in the interval between -4.55 and 3.02? Note: Integers are whole numbers and their opposites { … -4, -3, -2 , -1, 0 , 1 , 2 , 3 …}	A. 2 B. 3 C. 7 D. 8 E. Infinitely many
4. If p is a positive integer, then $p(p + 1)(p – 1)$ is always divisible by	A. 7 B. 5 C. 4 D. 3 E. None of the above
5. What is the smallest integer that is the sum of three different primes, each of which is greater than 10?	A. 41 B. 33 C. 12 D. 10 E. 6
6. If $a = (-4 + 24) \div (8 - 4)$ *and* $b = -4 + 24 \div 8 - 4$, then what is $\lvert a - b \rvert$?	A. -5 B. 0 C. 5 D. 10 E. None of the above

SHSAT Lesson #1: Homework (continued)

7. The product of two two-digit numbers *MN* and *NM* is equal to the three-digit number *NKN*. If *K*, *M*, and *N* are three different non-zero digits and the sum of *K* and *N* is 7, what is the two-digit number *MN*?	A. 31 B. 21 C. 13 D. 12 E. 11
8. If *D* and *C* are equal to 3 and 5, what is the value of $\|C - 2D\|$	A. 6 B. 5 C. 4 D. 3 E. None of the above
9. What is the smallest positive difference between two different prime numbers?	A. 0 B. 1 C. 2 D. 3 E. None of the above
10. The floor function $\lfloor x \rfloor$ *means* "the greatest integer less than x. What is the value of $(\lfloor 15.2 \rfloor - \lfloor 9.789 \rfloor)$ equal to ?	A. 10 B. 8 C. 6 D. 4 E. 2
11. Letter *A* substitutes a nonzero digit, and *AA* is a two-digit number. The product *AA* and *A* is always:	A. Even B. Divisible by 3 C. Divisible by 11 D. A perfect square E. Odd
12. *X*, *Y*, and *Z* are three consecutive whole numbers. Which of the following statements is always true?	A. *X* x *Y* x *Z* is even B. *X* x *Y* x *Z* is odd C. *X* + *Y* + *Z* is even D. *X* + *Y* + *Z* is odd E. None of the above

SHSAT Lesson #1: Homework (continued)

13. How many numbers between 12 and 27, inclusive, are divisible by 3?	A. 4 B. 5 C. 6 D. 8 E. 9
14. If M and N are 2 positive numbers and $M > N$, which of the following expressions is the smallest?	A. $-(M - N)$ B. $-M$ C. $-(N - M)$ D. $-(M + N)$ E. $-N$
15. The product of the first 5 prime numbers is divisible by Note: Prime numbers are divisible by 1 and themselves only {2,3,5,7,11,13…} are the first few primes.	A. 12 B. 16 C. 18 D. 24 E. 30
16. $24 \div (7 - 3)(2) + 2 + 20 \div 5 - 13 =$	A. -5 B. -4 C. -3 D. 13 E. None of the above
17. At 9AM it was 12 degrees below zero. By noon the temperature had dropped 7 degrees. Over the next two hours, the temperature rose 5 degrees. What was the temperature at 2 P.M.?	A. 0^o B. 10^o below zero C. 14^o below zero D. 24^o below zero E. None of the above
18. If T and P are non-zero integers with opposite signs, then which expressions must be positive?	A. $-5T^2P^2$ B. $-7TP$ C. $T + P^2$ D. T^2P E. $-2T + P^2$

SHSAT Lesson #1: SHSAT Exam Homework:

19. One-half of one-sixth is equal to	A. one-third of one-fourth B. one-third C. one D. two E. Three
20. Anne, Betty, and Charlie are teenagers, and the sum of their ages now is 51. The sum of their ages 7 years from now will be *F*, and the sum of their ages 11 years ago was *P*. The value of $F - P$ is	A. 54 B. 18 C. 17 D. 6 E. 2
21. When the integer N is divided by 7, the quotient is Q and the remainder is 5. When N+24 is divided by 7, the remainder is	A. 4 B. 3 C. 2 D. 1 E. 0
22. $\overline{.3} + \frac{1}{6} =$	A. .4 B. .6 C. $\frac{2}{3}$ D. $\frac{5}{6}$ E. $\frac{1}{2}$
23. The least common multiple of $2, 3, 4, 5, 6$ is	A. $2 * 3 * 4 * 5 * 6$ B. $2 * 3 * 4 * 5$ C. $2 * 3 * 5$ D. $3 * 4 * 5$ E. 2^5
24. $(6)(6) + (-6)(-6) =$	A. 0 B. 24 C. 72 D. 180 E. none of these

Name: **Date:**

Chang Learning	**SHSAT Lesson #1: Number Sense Part 1**

SHSAT Homework: Grid In Questions

25. If 2^{12} is multiplied out, what is the ones digit ?	Grid In
26. How many prime numbers are divisible by 11 ?	Grid In
27. $36 \div (12 - 3) * 2 - 15 \div 5 - 10 =$	Grid In
28. A stack of 5 books {red, orange, yellow, green & blue} are placed on a desk. If the bottom book must be red, how many ways can the 5 books be stacked ?	Grid In
29. From 7:00 P.M. to 8:00 P.M., Jose completed one-third of his homework. From 8:00 P.M. to 9:00 P.M., he completed one-half of the remaining part of his homework. What fraction of his homework still remained to be completed after 9:00 P.M.?	Grid In
30. Wei Jing has between 70 and 150 baseball cards in his collection. When he arranges them in groups of 10, he has 3 left over. When he arranges them in groups of 9, he has 5 left over. If he arranges them in groups of 8, how many will he have left over?	Grid In

Extra Credit Challenge Questions:

1. The number 695 is to be written with a factorial base of numeration, that is,

$$695 = (a_1) + (a_2)2! + (a_3)3! + \ldots + (a_n)n! \quad \text{where } a_1, a_2, \ldots a_n \text{ are integers such that } 0 \le a_k \le k,$$
$$\text{and } n! \text{ means } n(n-1)(n-2)\ldots(2)(1). \text{ Find } a_4.$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

2. If $8^x = 32$, then x equals:

(A) 4 (B) $\frac{5}{3}$ (C) $\frac{3}{2}$ (D) $\frac{3}{5}$ (E) $\frac{1}{4}$

3. A square and an equilateral triangle have equal perimeters. The area of the triangle is $9\sqrt{3}$ square inches. Expressed in inches, the diagonal of the square is

(A) 9/2 (B) $2\sqrt{5}$ (C) $4\sqrt{2}$ (D) $9\sqrt{2}/2$ (E) none of these

4. A gives B as many cents as B has and C as many cents as C has. Similarly, B then gives A and C as many cents as each then has. C, similarly, then gives A and B as many cents as each then has. If each finally has 16 cents, with how many cents does A start ?

(A) 24 (B) 26 (C) 28 (D) 30 (E) 32

5. A regular polygon of n sides is inscribed in a circle of radius R. The area of the polygon is $3R^2$. Then n equals:

(A) 8 (B) 10 (C) 12 (D) 15 (E) 18

MAA High School Contests 1960-1964 {61 #35, 62 #4, 62 #6, 63 #23, 61 #32}