

### SAT Lesson 3: Linear Equations

Linear equations are a fundamental equation for representing real world situations which have a relationship defined with a fixed and variable term. In our mathematics studies, we study direct variations of the form  $y = kx$  which has a fixed point at  $(0,0)$ . Linear equations expressed in “slope-intercept” form have an extra term  $\pm b$  attached to the equation. When graphed on the cartesian coordinate plane, they represent straight lines. Many SAT questions contain linear equations.

Slope-intercept	Standard form	Point-slope form
$y = mx + b$	$ax + by = c$	$(y - y_1) = m(x - x_1)$

<p style="text-align: center;"><b><u>Slope-Intercept Form</u></b></p> <p>The form is <math>y = mx + b</math>.</p> <p>This is a line with <b>slope</b> <math>m = -\frac{1}{3}</math> and <b>y-intercept</b> equal to <math>b = 6</math>.</p>	$y = -\frac{1}{3}x + 6$						
<p><b><u>Standard Form</u></b></p> <p>The form is <math>ax + by = c</math>.</p> <p>This is useful to plot with an intercept table.</p> <table border="1" style="margin: 10px auto; text-align: center;"> <tbody> <tr> <td style="padding: 5px;">X</td> <td style="padding: 5px;">Y</td> </tr> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">6</td> </tr> <tr> <td style="padding: 5px;">18</td> <td style="padding: 5px;">0</td> </tr> </tbody> </table> <p>This line has <b>x-intercept</b> at <math>(x, y) = (18, 0)</math> This line has <b>y-intercept</b> at <math>(x, y) = (0, 6)</math></p>	X	Y	0	6	18	0	$x + 3y = 18$
X	Y						
0	6						
18	0						
<p><b><u>Point-Slope Form</u></b></p> <p>The form is <math>(y - y_1) = m(x - x_1)</math>.</p> <p>A single point location on a line with a slope. For this line, there exists a <b>point</b> <math>(x_1, y_1) = (3, 5)</math> which is on a line with <b>slope</b> <math>m = -\frac{1}{3}</math>.</p>	$(y - 5) = -\frac{1}{3}(x - 3)$						

## Example: Linear Equation

Maria has a cell phone with a cost of only \$3.25 per month.  
However, she has to pay \$0.15 for each minute of use.  
The company then offers her a \$60 per month unlimited plan for all calls.

How many minutes of use per month makes it more  
affordable for Maria to pay for the unlimited plan ?

**Setup:**  $C(t) = \$3.25 + \$0.15t$

Since the unlimited plan is \$60, set the equation equal to sixty dollars.

$$C(t) = \$3.25 + \$0.15t$$

$$\$60 = \$3.25 + \$0.15t$$

$$\$56.75 = \$0.15t$$

$$\$56.75/\$0.15 = t$$

$$378 \frac{1}{3} = t$$

**Answer:**

**Maria will save money if she chooses the unlimited plan  
if she will use her phone for more than  $t > 378\frac{1}{3}$  minutes each month.**

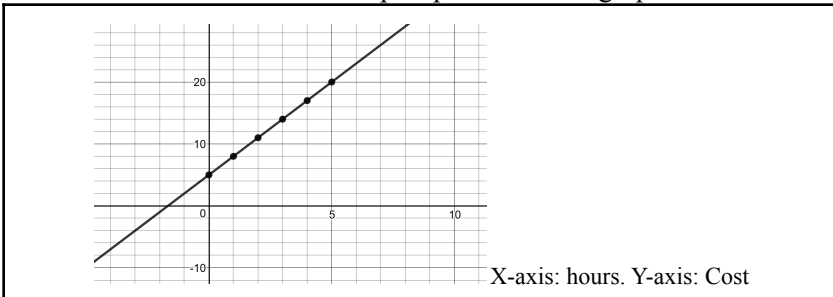
## SAT Lesson #3 Classwork: Linear Equations

1. In the equation $y = \frac{3}{5}x - 12$ , what is the slope ?	A) 1 B) $\frac{3}{5}$ C) $\frac{5}{3}$ D) $-7$
2. If $y = \frac{4}{7}x - 12$ , what is the y-intercept ?	A) 1 B) $-1$ C) 7 D) $-7$
3. For the line $y = \frac{3}{5}x - 7$ , what is the x-intercept ?	A) $\frac{35}{3}$ B) $-\frac{35}{3}$ C) $\frac{5}{3}$ D) $-\frac{5}{3}$
4. What is the slope of $(y - 6) = -\frac{2}{3}(x + 3)$ ?	A) $\frac{2}{3}$ B) $-\frac{2}{3}$ C) $\frac{3}{2}$ D) $-\frac{3}{2}$
5. What is the slope of the linear equation $(y + 2) = 3(x + 1)$ ?	A) 1 B) 2 C) 3 D) $-1$

## SAT Lesson #3: Classwork (continued)

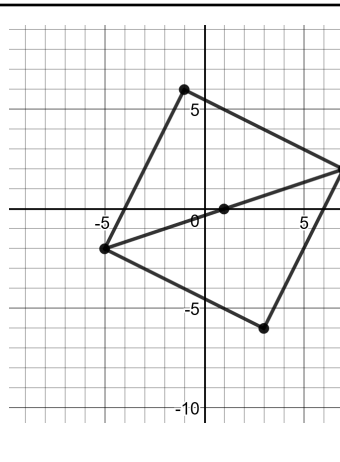
6. If $3r + 5 = 23$ , what is the value of $6r + 3$ ?	A) 6 B) 27 C) 36 D) 39
7. If $\frac{5}{x} = \frac{15}{x+20}$ , what is the value of $\frac{x}{5}$ ?	A) 10 B) 5 C) 2 D) $\frac{1}{2}$
8. Write $y = \frac{3}{5}x - 7$ in standard form.	A) $5x + 3y = -35$ B) $3x + 5y = -35$ C) $-3x + 5y = -35$ D) $-3x + 5y = 35$
9. What point $(x,y)$ lies on $(y - 6) = -\frac{3}{4}(x + 3)$ ?	A) (1, 9) B) (-3, -6) C) (-3, 6) D) (5, 14)
10. Express $(y + 2) = 3(x + 1)$ in slope intercept form.	A) $y = 3x + 1$ B) $y = 3x + 3$ C) $y = 3x + 5$ D) $y = 3x - 1$

## SAT Lesson #3: Classwork (continued)

<p>11. What is the y-intercept of the line <math>(y + 2) = 3(x + 1)</math> ?</p>	<p>A) <math>-1</math>            B) <math>5</math>            C) <math>3</math>            D) <math>1</math></p>
<p>12. Kaylie is a botanist studying the production of apples by two types of apple trees. She noticed that Type A tree produced 30 percent more apples than Type B trees did. Based on Kaylie's observation, if the Type A trees produced 156 apples, how many apples did the type B trees produce ?</p>	<p>A) 115            B) 120            C) 124            D) 173</p>
<p>13. A line in the xy-plane passes through the origin and has a slope of <math>\frac{1}{5}</math>. Which of the following points lies on the line ?</p>	<p>A) <math>(0, 5)</math>            B) <math>(1, 5)</math>            C) <math>(5, 5)</math>            D) <math>(15, 3)</math></p>
<p>14. The graph below displays the total cost <math>C</math>, in dollars, of renting a boat for <math>h</math> hours.            What does the C intercept represent in the graph ?</p> 	<p>A) The initial cost of renting the boat            B) The total number of boats rented            C) The total number of hours the boat is rented            D) The increase in cost to rent the boat for each additional hour</p>
<p>15. Which of the following represents the relationship between <math>h</math> and <math>C</math> ? (Use graph of previous question #17)</p>	<p>A) <math>C = 5h</math>            B) <math>C = \frac{3}{4}h + 5</math>            C) <math>C = 3h + 5</math>            D) <math>h = 3C</math></p>

## SAT Lesson #3: Classwork (continued)

16. Rewrite $3x + 4y = 15$ into point-slope form.	<p>A) <math>(y - 6) = -\frac{3}{4}(x + 3)</math>            B) <math>(y - 6) = -\frac{3}{4}(x - 3)</math>            C) <math>(y + 6) = -\frac{3}{4}(x + 3)</math>            D) <math>(y + 6) = -\frac{3}{4}(x - 3)</math></p>
17. Bart bought a laptop computer at a store that gave a 25 percent discount of its original price. The total amount she paid to the cashier was $p$ dollars, including an 6 percent sales tax on the discounted price. Which of the following represents the original price of the computer in terms of $p$ ?	<p>A) <math>0.81p</math>            B) <math>\frac{p}{0.81}</math>            C) <math>(0.75)(1.06)p</math>            D) <math>\frac{p}{(0.75)(1.06)}</math></p>
18. The graph of a line in the $xy - plane$ has slope 2 and contains the point $(1, 8)$ . The graph of a second line passes through the points $(1, 2)$ and $(2, 1)$ . If the two lines intersect at the point $(a, b)$ , what is the value of $a + b$ ?	<p>A) 4            B) 3            C) <math>-1</math>            D) <math>-4</math></p>
19. The graph of the linear function $f$ has intercepts at $(a, 0)$ and $(0, b)$ in the $xy - plane$ . If $a + b = 0$ and $a \neq b$ , which of the following is true about the slope of the graph of $f$ ?	<p>A) It is positive.            B) It is negative.            C) It equals zero.            D) It is undefined.</p>
<p>20. In the <math>xy - plane</math> below, ABCD is a square and point E is the center of the square. The coordinates of points C and E are <math>(7, 2)</math> and <math>(1, 0)</math>, respectively. Which of the following is an equation of the line that passes through points B and D ?</p>	<p>A) <math>y = -3x - 1</math>            B) <math>y = -3(x - 1)</math>            C) <math>y = -\frac{1}{3}x + 4</math>            D) <math>y = -\frac{1}{3}x - 4</math></p>



## SAT Lesson #3: Classwork SAT Review Grid-In (continued)

<p>21. <b>(Easy Level)</b> The table below lists the ages of the first 12 United States presidents when they began their terms in office. According to the table, what was the mean age, in years, of these presidents at the beginning of their terms? (Round your answer to the nearest tenth.)</p> <table border="1" data-bbox="203 527 1062 779"> <thead> <tr> <th>President</th> <th>Age</th> <th>President</th> <th>Age</th> <th>President</th> <th>Age</th> <th>President</th> <th>Age</th> </tr> </thead> <tbody> <tr> <td>Washington</td> <td>57</td> <td>Madison</td> <td>58</td> <td>Jackson</td> <td>62</td> <td>Tyler</td> <td>51</td> </tr> <tr> <td>Adams</td> <td>62</td> <td>Monroe</td> <td>59</td> <td>Van Buren</td> <td>55</td> <td>Polk</td> <td>50</td> </tr> <tr> <td>Jefferson</td> <td>58</td> <td>Adams</td> <td>58</td> <td>Harrison</td> <td>68</td> <td>Taylor</td> <td>65</td> </tr> </tbody> </table>	President	Age	President	Age	President	Age	President	Age	Washington	57	Madison	58	Jackson	62	Tyler	51	Adams	62	Monroe	59	Van Buren	55	Polk	50	Jefferson	58	Adams	58	Harrison	68	Taylor	65	Grid-In
President	Age	President	Age	President	Age	President	Age																										
Washington	57	Madison	58	Jackson	62	Tyler	51																										
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Jefferson	58	Adams	58	Harrison	68	Taylor	65																										
<p>22. <b>(Easy Level)</b> Maria plans to rent a boat. The boat rental costs \$60 per hour, and she will also have to pay for a water safety course that costs \$10. Maria wants to spend no more than \$280 for the rental and the course. If the boat rental is available only for a whole number of hours, what is the maximum number of hours for which Maria can rent the boat?</p>	Grid-In																																
<p>23. <b>(Mid Level)</b> A local television station sells time slots for programs in 30 minute intervals. If the station operates 24 hours per day, every day of the week, what is the total number of 30 minute time slots the station can sell for Tuesday and Wednesday?</p>	Grid-In																																
<p>24. <b>(Mid Level)</b> The normal systolic blood pressure, <math>P</math>, in millimeters of mercury, for an adult male <math>x</math> years old can be modeled by the equation <math>P = \frac{x+220}{2}</math>. According to the model, for every increase of 1 year in age, by how many millimeters of mercury will the normal systolic blood pressure for an adult male increase</p>	Grid-In																																
<p>25. <b>(Challenge Level)</b> The mesosphere is the layer of Earth's atmosphere between 50 kilometers and 85 kilometers above Earth's surface. At a distance of 50 kilometers from Earth's surface, the temperature in the mesosphere is <math>-5^{\circ}</math>Celsius, and at a distance of 80 kilometers from the Earth's surface, the temperature in the mesosphere is <math>-80^{\circ}</math>Celsius. For every additional 10 kilometers from the Earth's surface, the temperature in the mesosphere decreases by <math>k^{\circ}</math>Celsius, where <math>k</math> is a constant. What is the value of <math>k</math>?</p>	Grid-In																																

## SAT Lesson #3 Homework: Linear Equations

1. In the equation $y = \frac{1}{3}x + 9$ , what is the slope ?	A) 1 B) $\frac{2}{3}$ C) $\frac{1}{3}$ D) 9
2. If $y = \frac{1}{7}x - 3$ , what is the y-intercept ?	A) $\frac{1}{2}$ B) 2 C) 1 D) - 3
3. For the line $y = \frac{1}{2}x - 3$ , what is the x-intercept ?	A) - 3 B) 6 C) - 6 D) $\frac{1}{2}$
4. What is the slope of $(y - 10) = -\frac{5}{9}(x + 6)$ ?	A) $\frac{5}{9}$ B) $-\frac{5}{9}$ C) $\frac{9}{5}$ D) $-\frac{9}{5}$
5. What is the slope of the linear equation $(y - 9) = \frac{9}{16}(x - 6)$ ?	A) $\frac{16}{9}$ B) $\frac{9}{16}$ C) $-\frac{9}{16}$ D) $-\frac{16}{9}$



## SAT Lesson #3: Homework (continued)

<p>6. The table below shows some values of the linear function <math>f</math>. Which of the following defines <math>f</math>?</p> <table border="1" data-bbox="363 380 833 512"> <tbody> <tr> <td>n</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td><math>f(n)</math></td> <td>-2</td> <td>1</td> <td>4</td> <td>7</td> </tr> </tbody> </table>	n	1	2	3	4	$f(n)$	-2	1	4	7	<p>A) <math>f(n) = n - 3</math>            B) <math>f(n) = 2n - 4</math>            C) <math>f(n) = 3n - 5</math>            D) <math>f(n) = 4n - 6</math></p>
n	1	2	3	4							
$f(n)$	-2	1	4	7							
<p>7. Josh earns \$15 per hour working for a local restaurant. In addition to his base salary, for each delivery he makes, the store pays him a \$10 delivery fee. How much will he earn if he works 30 hours this week and also delivers 12 food deliveries?</p>	<p>A) \$450            B) \$570            C) \$600            D) \$640</p>										
<p>8. Nathaniel walks 25 meters in 13.7 seconds. If he walks at this same rate, which of the following is closest to the distance he will walk in 4 minutes?</p>	<p>A) 150 meters            B) 450 meters            C) 700 meters            D) 1,400 meters</p>										
<p>9. The cost of using a telephone in a private business meeting room is \$0.30 per minute. Which of the following equations represents the total cost <math>c</math>, in dollars, for <math>h</math> hours of phone use?</p>	<p>A) <math>c = 0.30(60h)</math>            B) <math>c = 0.30(60h)</math>            C) <math>c = \frac{60h}{0.30}</math>            D) <math>c = \frac{0.30h}{60}</math></p>										
<p>10. Write <math>y = \frac{1}{2}x - 3</math> in standard form.</p>	<p>A) <math>x + 2y = 6</math>            B) <math>x - 2y = 6</math>            C) <math>-x + 2y = 6</math>            D) <math>-x - 2y = 6</math></p>										

## SAT Lesson #3: Homework (continued)

<p>11. Rewrite <math>5x + 6y = 30</math> into point-slope form.</p>	<p>A) <math>(y - 10) = -\frac{5}{6}(x + 6)</math>            B) <math>(y - 10) = -\frac{5}{6}(x - 6)</math>            C) <math>(y + 10) = -\frac{5}{6}(x + 6)</math>            D) <math>(y + 10) = -\frac{5}{6}(x - 6)</math></p>												
<p>12. What point <math>(x,y)</math> lies on   <math>(y - 10) = -\frac{5}{6}(x + 6)</math> ?</p>	<p>A) <math>(+ 1, + 9)</math>            B) <math>(- 6, - 10)</math>            C) <math>(- 6, + 10)</math>            D) <math>(+ 6, - 10)</math></p>												
<p>13. What is the y-intercept of the line   <math>(y - 9) = \frac{9}{16}(x - 6)</math> ?</p>	<p>A) <math>\frac{45}{8}</math>            B) <math>\frac{29}{8}</math>            C) <math>\frac{13}{8}</math>            D) <math>- 1</math></p>												
<p>14. The function <math>f</math> is defined by a polynomial. Since values of <math>x</math> and <math>f(x)</math> are shown in the table below. Which of the following must be a factor of <math>f(x)</math> ?</p> <table border="1" data-bbox="298 1171 906 1306"> <tbody> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>y</td> <td>-2</td> <td>0</td> <td>4</td> <td>10</td> <td>18</td> </tr> </tbody> </table>	x	1	2	3	4	5	y	-2	0	4	10	18	<p>A) <math>x - 2</math>            B) <math>x - 3</math>            C) <math>x - 4</math>            D) <math>x - 5</math></p>
x	1	2	3	4	5								
y	-2	0	4	10	18								
<p>15. The sum of three numbers is 855. One of the numbers, <math>x</math>, is 50% more than the sum of the other two numbers. What is the value of <math>x</math> ?</p>	<p>A) 570            B) 513            C) 214            D) 155</p>												

## SAT Lesson #3: Homework (continued)

<p>16. The average number of students per classroom at Central High School from 2000 to 2010 can be modeled by the equation <math>y = 0.56x + 27.2</math>, where <math>x</math> represents the number of years since 2000, and <math>y</math> represents the average number of students per classroom. Which of the following best describes the meaning of the number 0.56 in the equation ?</p>	<p>A) The total number of students at the school in 2000            B) The average number of students per classroom in 2000            C) The estimated increase in the average number of students per classroom each year            D) The estimated difference between the average students per classroom in 2010 &amp; 2000</p>
<p>17. The equation below shows how a to convert fahrenheit to celsius. Based on the equation, which of the following is true ?</p> <div style="border: 1px solid black; padding: 10px; width: fit-content; margin: 10px auto;"> <math display="block">C = \frac{5}{9} (F - 32)</math> </div> <p>I. <math>32F = 0C</math>      II. <math>212F = 100C</math>      III. <math>68F = 20C</math></p>	<p>A) I, II and III            B) II only            C) III only            D) I and II only</p>
<p>18. Express <math>(y - 9) = \frac{9}{16} (x - 6)</math> in slope intercept form.</p>	<p>A) <math>y = \frac{9}{16}x + 5\frac{5}{8}</math>            B) <math>y = \frac{9}{16}x + 3\frac{5}{8}</math>            C) <math>y = \frac{9}{16}x + 1\frac{5}{8}</math>            D) <math>y = \frac{9}{16}x + 3</math></p>
<p>19. The line <math>y = kx + 4</math>, where <math>k</math> is a constant, is graphed in the <math>xy - plane</math>. If the line contains the point <math>(c, d)</math>, where <math>c \neq 0</math> and <math>d \neq 0</math>, what is the slope of the line in terms of <math>c</math> and <math>d</math> ?</p>	<p>A) <math>\frac{d-4}{c}</math>            B) <math>\frac{c-4}{d}</math>            C) <math>\frac{4-d}{c}</math>            D) <math>\frac{4-c}{d}</math></p>
<p>20. Which of the following is an example of a function whose graph in the <math>xy - plane</math> has no <math>x - intercepts</math> ?</p>	<p>A) A linear function whose rate of change is not zero            B) A quadratic function with real zeros            C) A quadratic function with no real zeros            D) A cubic polynomial with at least one real zero</p>

## SAT Lesson #3: Homework SAT Review Grid-In (continued)

<p>21. <b>(Easy Level)</b></p> <p>What value of <math>t</math> is the solution of the equation <math>\frac{2}{3}t = \frac{5}{2}</math> ?</p>	Grid-In:
<p>22. <b>(Easy Level)</b></p> <p>Horsepower and watts are units of measure of power. They are directly proportional such that 5 horsepower is equal to 3730 watts. How much power, in watts, is equal to 2 horsepower ?</p>	Grid-In:
<p>23. <b>(Mid Level)</b></p> <p>If <math>a^{\frac{b}{4}} = 16</math> for positive integers <math>a</math> and <math>b</math>, what is one possible value of <math>b</math> ?</p>	Grid-In:
<p>24. <b>(Mid Level)</b></p> <p>In a study of bat migration habits, 240 male bats and 160 female bats have been tagged. If 100 more female bats are tagged, how many more male bats must be tagged so that <math>\frac{3}{5}</math> of the total number of bats in the study are male ?</p>	Grid-In:
<p>25. <b>(Challenge Level)</b></p> <p>A group of friends decided to divide the \$800 cost of a trip equally among themselves. When two of the friends decided not to go on the trip, those remaining still divided the \$800 cost equally, but each friend's share of the cost increased by \$20. How many friends were in the group originally ?</p>	Grid-In:

## Extra Credit Challenge Questions:

## SAT Level 1

1. A circle has circumference of $16\pi$ cm. What is its area ?	A. $8\pi \text{ cm}^2$ B. $16\pi \text{ cm}^2$ C. $332\pi \text{ cm}^2$ D. $64\pi \text{ cm}^2$ E. $256\pi \text{ cm}^2$
2. Assuming $x \neq 0$ , $\frac{1}{(x/3)^2} =$	A. $\frac{x^2}{3}$ B. $\frac{1}{3x^2}$ C. $\frac{x^2}{9}$ D. $\frac{9}{x}$ E. $\frac{9}{x^2}$
3. In triangle $\Delta XYZ$ we have that $XY = YZ$ . If the measure of <i>angle Y</i> is $50^\circ$ , what is the measure of <i>angle Z</i> ?	A. $50^\circ$ B. $130^\circ$ C. $65^\circ$ D. $75^\circ$ E. $25^\circ$
4. A cone and a cylinder both have height $h$ and a radius $r$ . If the volume of the cone is $12\pi \text{ cm}^3$ , what is the volume of the cylinder ?	A. $4\pi \text{ cm}^3$ B. $12\pi \text{ cm}^3$ C. $24\pi \text{ cm}^3$ D. $36\pi \text{ cm}^3$ E. $48\pi \text{ cm}^3$
5. What is the measure of the angle formed by the hands of a clock at 5 o'clock ?	A. $120^\circ$ B. $140^\circ$ C. $150^\circ$ D. $160^\circ$ E. $170^\circ$

SAT Level 1: Answers: [1]D [2]E [3]A [4]D [5]C

**Extra Credit Challenge Questions:**
**SAT Level 2**

<p>1. In the <math>xy</math>-plane, which of the following is an equation for the line that contains the point <math>(2, -3)</math> and has an <math>x</math>-intercept of 5 ?</p>	<p>A. <math>y = -4x + 5</math>            B. <math>y = x - 5</math>            C. <math>y = 2x - 7</math>            D. <math>y = 3x - 9</math>            E. <math>y = 5x - 3</math></p>
<p>2. If <math>f(x) = x^2 - 1</math>, which of the following are true ?</p> <p>I. <math>f(x) = f(-x)</math> for all <math>x</math>.            II. <math>f(x) = -f(x)</math> for all <math>x</math>.            III. <math>f(0) &gt; f(-1)</math>.</p>	<p>A. I only            B. III only            C. I and II only            D. II and III only            E. I, II, and III</p>
<p>3. If <math>f(x) = \ln(x)</math> and <math>g(x) = 2x + 1</math>, then <math>f(g(\sqrt{2})) =</math></p>	<p>A. 1.32            B. 1.34            C. 1.69            D. 1.73            E. 2.04</p>
<p>4. <math>\{1, 5, 10, 100\}</math></p> <p>If two distinct numbers are chosen at random from the set above, what is the probability that their sum will be greater than 100 ?</p>	<p>A. 0.17            B. 0.33            C. 0.50            D. 0.67            E. 0.83</p>
<p>5. In 2004 there were approximately 293 million people living in the United States. The population was increasing by approximately 0.92 percent per year. If the population continues to increase at the same rate, approximately how many people will be living in the United States in 2014 ?</p>	<p>A. 127            B. 296            C. 321            D. 333            E. 706</p>

**SAT Level 2: Answers: [1]B [2]A [3]B [4]C [5]C**