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Chang Learning	**SAT Lesson 9: Quadratic Formulas**

Quadratic Formulas

In the previous lesson, we worked on a variety of SAT skills to factor quadratics, many of which start in standard form $f(x) = ax^2 + bx + c$. Some students find it more useful to solve for the zeros, or roots, of a parabola by using the quadratic formula. This lesson will cover topics with the quadratic formula including solving for zeros, axis of symmetry and the nature of the roots. Also it will discuss the important alternate form from classical mathematics called "vertex form" which locates the turning point of the parabola as graphed on the Cartesian coordinate plane.

Example 1: Using the Quadratic Formula $f(x) = ax^2 + bx + c$.

What are the zero solutions to $f(x) = x^2 + 3x - 10$ **?**

Step 1: Set the quadratic in degree order equal to zero.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 + 3x - 10 = 0$$

$a = 1$	$b = 3$	$c = -10$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-10)}}{2(1)} = \frac{-3 \pm \sqrt{49}}{2} = \frac{-3 \pm 7}{2}$$

The equation $f(x) = x^2 + 3x - 10$ has zeros at -5 *and* $+2$.

It's important to realize that the terms of the quadratic formula can tell us a lot about the function even before we graph it on the xy-plane. There are three detailed questions about parabolas that are commonly tested on the SAT: Firstly, parabolas are symmetric and have a symmetry line. Secondly, they have zero, one or two roots along the x-axis, depending on their location on the plane. Thirdly, they have a turning point, or vertex.

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Example 2: The Discriminant: Nature of the Roots

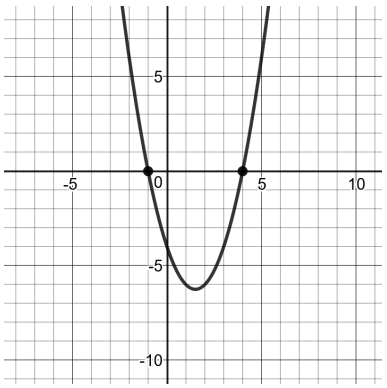
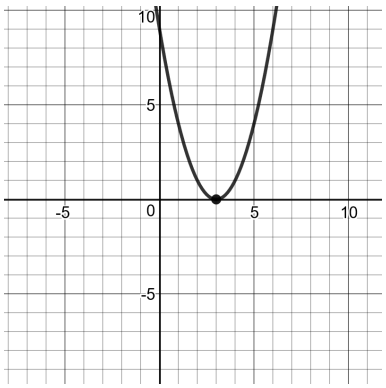
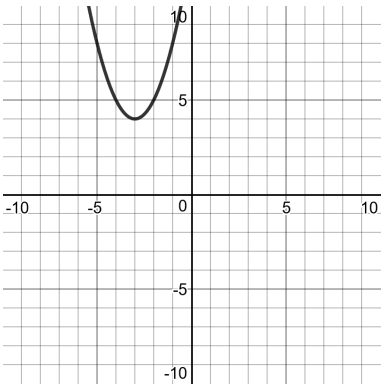
https://www.youtube.com/watch?v=IKqXu-5jw60

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The square root term of the quadratic formula is called "the discriminant". This term tells us whether a graph, or locus of points, crosses the x-axis. These points are called zeros.

When the discriminant $\sqrt{b^2 - 4ac} > 0$, has two, real, rational roots.
When the discriminant $\sqrt{b^2 - 4ac} = 0$, has one, real, double root.
When the discriminant $\sqrt{b^2 - 4ac} < 0$, has two, imaginary, conjugate pair roots.

Here are some examples below:
The real zeros are graphed, but imaginary conjugate pairs do not exist on the xy-plane.

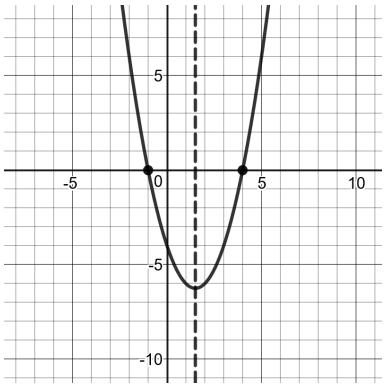
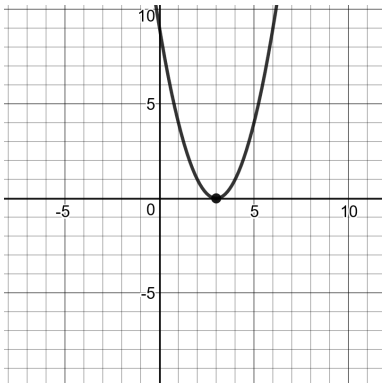
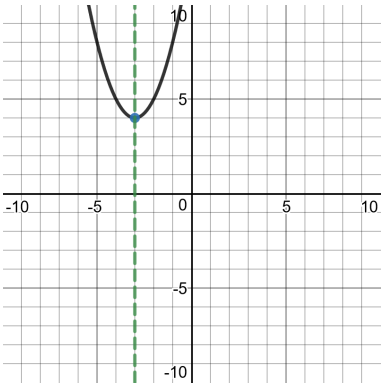
$y = x^2 - 3x - 4$	$y = x^2 - 6x + 9$	$y = x^2 + 6x + 13$
		
$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-4}}{2(1)}$	$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$	$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(13)}}{2(1)}$
Zeroes: $x = \{-1, 4\}$	Zeroes: $x = \{3\}$	Zeroes: $x = \{-3 \pm 2i\}$
Two Real Roots	One Real Root	Imaginary Conjugate Pair Roots

Example 3: Axis of Symmetry & Location of the Vertex

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The rational terms of the quadratic formula are useful for finding the centerline of the parabola, called "the axis of symmetry". This ratio informs us of the correct location of the axis on the xy-plane. points, crosses the x-axis.

$$x = \frac{-b}{2a}$$

$y = x^2 - 3x - 4$	$y = x^2 - 6x + 9$	$y = x^2 + 6x + 13$
		
$x = \frac{-(-3)}{2(1)}$	$x = \frac{-(-6)}{2(1)}$	$x = \frac{-6}{2(1)}$
Axis: $x = \{1.5\}$ Vertex: $y = 1.5^2 - 3(1.5) - 4$	Axis: $x = \{3\}$ Vertex: $y = 3^2 - 6(3) + 9$	Axis: $x = \{-3\}$ Vertex: $y = -3^2 + 6(-3) + 13$
The axis of symmetry is the line located at $x = 1.5$	The axis of symmetry is the line located at $x = 3$	The axis of symmetry is the line located at $x = -3$
The vertex is located at the point $(1.5, -6.25)$	The vertex is located at the point $(3, 0)$	The vertex is located at the point $(-3, 4)$

Example 4: Vertex form of the parabola

$$y = a(x - h)^2 + k$$

In classical mathematics lessons on Conics, the parabola is defined as the locus of points equidistant from a point of focus and a line called the directrix. In science, lenses are used to focus light, such as car headlights, train and plane lights. Radio signals and satellite signals are collected from the atmosphere and focused by parabolic receiver dishes.

An equivalent form to $y = ax^2 + bx + c$, the vertex form is useful in that the vertex point (h, k) is a part of the equation itself. The distance from the vertex to both the focus and directrix are the same measure, and are noted as distance "p".

What are the vertex form of the parabola $y = 0.5x^2 - 5x + 14.5$ **?**
The method we use to convert the equation from standard to vertex form is called "completing the square".

$$y = 0.5x^2 - 5x + 14.5$$

$$y = (0.5x^2 - 5x + \quad) + 14.5$$

$$y = 0.5(x^2 - 10x + \quad) + 14.5$$

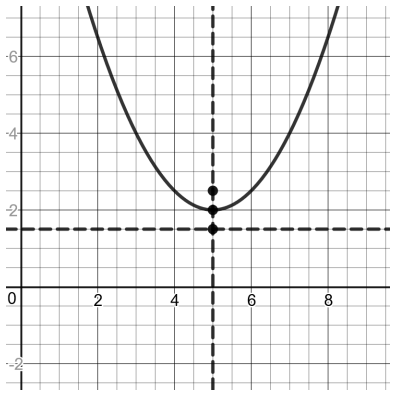
$$y = 0.5(x^2 - 10x + 25) - 0.5(25) + 14.5$$

$$y = 0.5(x - 5)^2 - 12.5 + 14.5$$

$$y = 0.5(x - 5)^2 + 2$$

Set the equation in standard form. Complete the square of the trinomial by first factoring out the a term. The "c" term of the trinomial will be $\left(\frac{b}{2}\right)^2$ **which is** $\left(\frac{10}{2}\right)^2 = 25.$ **Complete the square and be sure to subtract the c term after the parenthesis to keep balance. It's** $ac = 0.5(25)$ **that is then subtracted.**

Vertex at $(h, k) = (5, 2)$	**Focal Distance (p)** $a = \frac{1}{4p}$ $0.5 = \frac{1}{4p}$ $p = 0.5$



$y = 0.5(x - 5)^2 + 2$
Vertex location $(h, k) = (5, 2)$
Focus point is $p = 0.5$ **above at** $(5, 2.5)$
Directrix line is $p = 0.5$ **below at** $y = 1.5$

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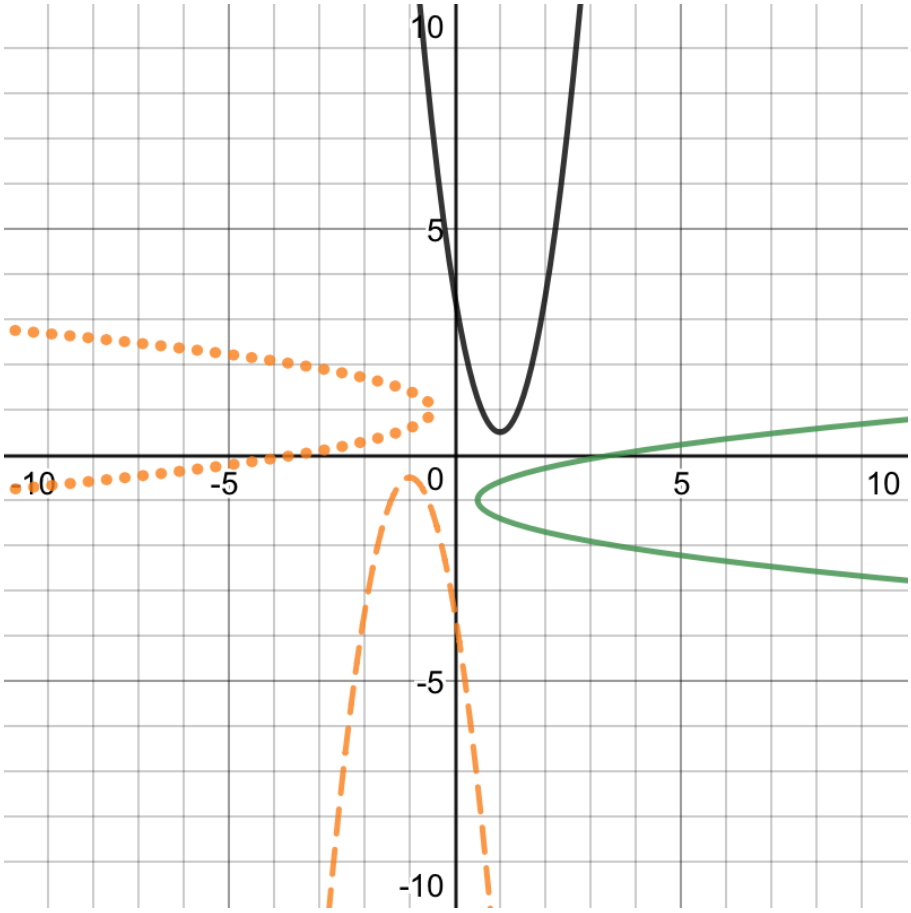
Example 5: Parabolas in 4 Directions

Final note: Most SAT questions with quadratic functions have parabolas facing either up or down in direction. A rare few have equations with parabola relations opening either right or left. This is done by "switching x & y" variables in the general form of the equation and also swapping "h & k" in the location of the vertex.

$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$



$y = \quad + 3(x - 1)^2 + 0.5$ **(Up)**	$x = \quad + 3(y + 1)^2 + 0.5$ **(Right)**
$y = \quad - 3(x + 1)^2 - 0.5$ **(Down)**	$x = \quad - 3(y - 1)^2 - 0.5$ **(Left)**

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SAT Lesson #9 Classwork: Quadratic Formulas

1) If $y = x^2 - 8x + 15$, for which values does $y = 0$?	A) $\{1, 3\}$ B) $\{3, 5\}$ C) $\{-1, 3\}$ D) $\{3, -5\}$
2) Where does the parabola $y = 6x^2 + x - 2$ cross the x-axis?	A) $+\frac{1}{2}, \ +\frac{2}{3}$ B) $-\frac{1}{2}, \ +\frac{2}{3}$ C) $-\frac{1}{2}, \ -\frac{2}{3}$ D) $+\frac{1}{2}, \ -\frac{2}{3}$
3) What are the x-intercepts of $y + \frac{7}{20} = x^2 - \frac{1}{5}x$?	A) $(+0.5, 0)$ *and* $(+0.7, 0)$ B) $(-0.5, 0)$ *and* $(+0.7, 0)$ C) $(0, -0.35)$ *and* $(+0.7, 0)$ D) $(-0.5, 0)$ *and* $(-0.7, 0)$
4) When does the function $f(t) = 3t^2 - 42t + 147$ cross the line $f(t) = 12$?	A) 0 and 7 B) 2 and 7 C) 5 and 9 D) 7 and 11
5) What are the zero solutions to $y = x^2 - x - 28$?	A) $\frac{1 \pm \sqrt{88}}{2}$ B) $\frac{-1 \pm \sqrt{88}}{2}$ C) $\frac{1 \pm \sqrt{113}}{2}$ D) $\frac{28 \pm \sqrt{113}}{2}$

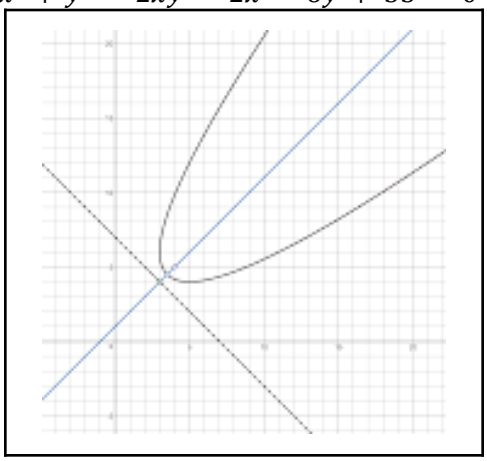
SAT Lesson #9: Classwork (Continued)

6)	For what values of y is $y = x^2 + 24x + 144$ zero?	A) $12\ and\ 0$ B) $-12\ and\ 0$ C) $-12\ only$ D) $12\ only$
7)	What is the nature of the roots of $y = -x^2 - 6x - 9$?	A) Two rational roots. B) One rational integer root. C) Two irrational roots. D) Two complex conjugate roots.
8)	What are the roots of $y = x^2 - (2\sqrt{3})x + 3$?	A) Two real roots B) Two irrational roots C) Two complex roots D) One irrational root
9)	What are the roots of the quadratic $f(x) = x^2 - 4x + 17$?	A) $2 \pm i\sqrt{13}$ B) $2 \pm 2i\sqrt{13}$ C) $2 \pm 4i\sqrt{13}$ D) $2 \pm i\sqrt{52}$
10)	What are the zeroes of $y = x^2 + 15$?	A) $\pm i\sqrt{15}$ B) $4 \pm 2i\sqrt{15}$ C) $2 \pm i\sqrt{15}$ D) $i \pm \sqrt{15}$

SAT Lesson #9: Classwork (Continued)

11)	What is the axis of symmetry of $y = x^2 + 5$?	A) $x = 0$ B) $x = 5$ C) $y = 0$ D) $y = 5$
12)	What is the axis of symmetry of $x = -3y^2 + 18y + 1$	A) $y = -3$ B) $y = -2$ C) $y = 2$ D) $y = 3$
13)	What is the symmetry line of $f(x) = (x + 1)(x - 5)$	A) $x = 1$ B) $x = 2$ C) $x = 3$ D) $x = 4$
14)	What is the axis of symmetry of $y - 5 = (x - 3)^2$	A) $x = 1.5$ B) $x = 3$ C) $x = 4$ D) $x = 5$
15)	What is the shortest distance from the line $y = 30$ to the equation $y = -3(x - 2)^2 + 15$?	A) 30 B) 20 C) 15 D) 12

SAT Lesson #9: Classwork (Continued)

16)	What is the location of the vertex of $f(x) = x^2 + 2$?	A) $(2, 6)$ B) $(0, 2)$ C) $(2, 0)$ D) $(0, 4)$
17)	What is the location of the vertex of $f(x) = -0.25(x - 1)^2 + 8$?	A) $(0.25, 8)$ B) $(-0.25, 8)$ C) $(-1, 8)$ D) $(1, 8)$
18)	What is the location of the vertex of $f(y) = (y - 4)^2 + 5$?	A) $(-4, 5)$ B) $(4, -5)$ C) $(-5, -4)$ D) $(5, 4)$
19)	What is the location of the vertex of $f(y) = y^2 + 4y + 3$?	A) $(-1, -2)$ B) $(1, -2)$ C) $(-2, -1)$ D) $(2, -1)$
20)	The parabola I wrote is symmetric on the line $y = x + 1$, has focus at $(4, 5)$, vertex at $(\frac{7}{2}, \frac{9}{2})$ and directrix line of $y = -x + 7$. It has the following "conic" formula: $x^2 + y^2 - 2xy - 2x - 6y + 33 = 0$  Which point is NOT on the parabola ?	A) $(3, 6)$ B) $(5, 12)$ C) $(11, 6)$ D) $(10, 18)$

SAT Lesson #9 Classwork SAT Grid-In Questions

21)	**(Easy Level)** On April 18th, 1775, Paul Revere set off on his midnight ride from Charlestown to Lexington. If he had ridden straight to Lexington without stopping, he would have travelled 11 miles in 26 minutes. In such a ride, what would the average speed of his horse have been, to the nearest tenth of a mile per hour ?	Grid In
22)	**(Easy Level)** The number of radians in a 720-degree angle can be written as $a\pi$, where a is a constant. What is the value of a ?	Grid In
23)	**(Mid Level)** Andrew and Maria each collected six rocks, and the masses of the rocks are shown in the table above. The mean of the masses of the rocks Maria collected is 0.1 kilogram greater than the mean of the masses of the rocks Andrew collected. What is the value of x ?	Grid In
24)	**(Mid Level)** The line PQRS has $PQ = x - 1,\ QR = x\ and\ RS = 3x - 7.$ PS as described as written also has PQ=RS, then what is the length of PS ?	Grid In
25)	**(Challenge Level)** In the xy-plane, the point (2,5) lies on the graph of the function f. If $f(x) = k - x^2$, where k is a constant, what is the value of k ?	Grid In

Andrew	2.4	2.5	3.6	3.1	2.5	2.7
Maria	x	3.1	2.7	2.9	3.3	2.8

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SAT Lesson #9 Homework: Quadratic Formulas

1) If $y = 2x^2 - 4x - 30$, for which values does $y = 0$?	A) $\{3, 5\}$ B) $\{-1, -5\}$ C) $\{-3, 5\}$ D) $\{3, -5\}$
2) Where does the parabola $y = \frac{1}{3}x^2 - \frac{2}{3}x - \frac{8}{3}$ cross the x-axis?	A) $\{1, 2\}$ B) $\{-2, 4\}$ C) $\{2, 4\}$ D) $\{6, 12\}$
3) When does the function $f(t) = 3t^2 - 42t + 147$ cross the line $f(t) = 27$?	A) 1 and 8 B) 2 and 12 C) 3 and 11 D) 4 and 10
4) For what values of y is $y = x^2 + 24x + 44$ zero?	A) $22\ and\ 2$ B) $-22\ and\ -2$ C) $-22\ only$ D) $-2\ only$
5) What are the zero solutions to $y = x^2 + 7x + 3$?	A) $\frac{-7 \pm \sqrt{37}}{2}$ B) $\frac{-7 \pm \sqrt{61}}{2}$ C) $\frac{-7 \pm \sqrt{52}}{2}$ D) $\frac{7 \pm \sqrt{37}}{2}$

SAT Lesson #9: Homework (Continued)

6)	What is the nature of the roots of $y = -x^2 - 6x - 10$?	A) Two rational roots. B) One rational integer root. C) Two irrational roots. D) Two complex conjugate roots.
7)	What are the x-intercepts of $y - 4 = 5x^2 - 11x$?	A) $\frac{-11 \pm \sqrt{41}}{10}$ B) $\frac{11 \pm \sqrt{41}}{10}$ C) $\frac{-11 \pm \sqrt{41}}{5}$ D) $\frac{11 \pm \sqrt{41}}{5}$
8)	What are the roots of the quadratic $f(x) = x^2 + 2x + 12$?	A) $1 \pm i\sqrt{11}$ B) $2 \pm i\sqrt{11}$ C) $-1 \pm i\sqrt{11}$ D) $-2 \pm i\sqrt{11}$
9)	What are the roots of $y = x^2 - (6\sqrt{2})x + 18$?	A) Two real roots B) Two irrational roots C) Two complex roots D) One irrational root
10)	What are the zeroes of $y = x^2 + 14$?	A) $\pm i\sqrt{14}$ B) $2 \pm 2i\sqrt{14}$ C) $1 \pm i\sqrt{14}$ D) $i \pm \sqrt{14}$

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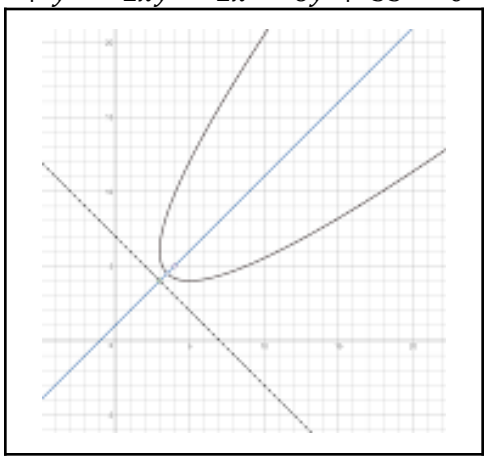
SAT Lesson #9: Homework (Continued)

11)	What is the axis of symmetry of $y = x^2 + 15$?	A) $x = 0$ B) $x = 15$ C) $y = 0$ D) $y = 15$
12)	What is the axis of symmetry of $x = -y^2 + 6y + 1$	A) $y = -3$ B) $y = -2$ C) $y = 3$ D) $y = 6$
13)	What is the symmetry line of $f(x) = (x + 5)(x - 1)$	A) $x = -1$ B) $x = -2$ C) $x = -3$ D) $x = -4$
14)	What is the axis of symmetry of $y + 2 = (x - 7)^2$	A) $x = 3.5$ B) $x = 5$ C) $x = 7$ D) $x = 9$
15)	What is the shortest distance from the line $y = 24$ to the equation $y = -3(x - 2)^2 + 12$?	A) 30 B) 20 C) 15 D) 12

SAT Lesson #9: Homework (Continued)

16)	What is the location of the vertex of $f(x) = -3x^2 + 4$?	A) $(2, 6)$ B) $(0, 2)$ C) $(2, 0)$ D) $(0, 4)$
17)	What is the location of the vertex of $f(x) = -9(x - 11)^2 + 15$?	A) $(11, 15)$ B) $(-11, 15)$ C) $(-11, -15)$ D) $(11, -15)$
18)	What is the location of the vertex of $f(y) = (y - 4)^2 + 5$?	A) $(-4, 5)$ B) $(4, -5)$ C) $(-5, -4)$ D) $(5, 4)$
19)	What is the location of the vertex of $f(y) = y^2 + 4y + 3$?	A) $(-1, -2)$ B) $(1, -2)$ C) $(-2, -1)$ D) $(2, -1)$
20)	The parabola I wrote is symmetric on the line $y = x + 1$, has focus at $(4, 5)$, vertex at $(\frac{7}{2}, \frac{9}{2})$. It has the following "conic" formula: $x^2 + y^2 - 2xy - 2x - 6y + 33 = 0$  Which point is NOT on the directrix line ?	A) $(6, 1)$ B) $(2, 5)$ C) $(-1, 9)$ D) $(0, 7)$

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SAT Lesson #9 Homework SAT Grid-In Questions

21)	**(Easy Level)** $(24y^2 + 125) + (75 - 12y^2)$ The expression shown can be written in the form $ay^2 + b,\ where\ a\ and\ b$ are constants. What is the value of $a + b$?	Grid In
22)	**(Easy Level)** A start-up company opened with 8 employees. The company's growth plan assumes that 2 new employees will be hired each quarter (every 3 months) for the first 5 years. If an equation is written in the form $y = ax + b$ to represent the number of employees, y, employed by the company x quarters after the company opened, what is the value of b ?	Solution:
23)	**(Mid Level)** The graph of the function f, defined by $f(x) = -\frac{1}{2}(x - 4)^2 + 10$. If the function $g(x) = -x + 10$, what is one possible value of a such that $f(a) = g(a)$?	Grid In
24)	**(Mid Level)** Point A and B lie on a circle with radius 1, and arc AB has length $\frac{\pi}{3}$. What fraction of the circumference of the circle is the length of arc AB ?	Grid In
25)	**(Challenge Level)** If black and white copies cost \$0.10 and Color Copies cost \$0.25, then what is the cost of 180 Color Copies and 120 Black & White copies, including 8% state tax ?	Grid In