

## Pythagorean Theorem

By now you have probably heard of the Pythagorean Theorem in your studies of mathematics. In 300 BCE, the mathematician Euclid wrote a 13 volume work on the subject that included the famous theorem. In this collection of geometric axioms and postulates, he lays the foundation for two and three dimensional geometry.

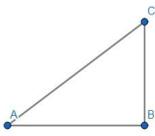
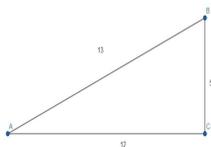
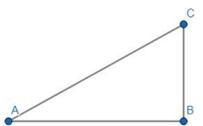
**The Pythagorean Theorem:**

**In right angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.**

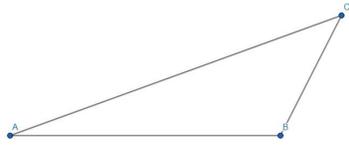
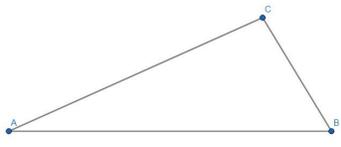
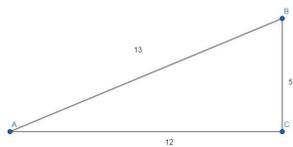
Euclid's Elements. Book I, Proposition 47. (300 BCE)

However, perhaps this important historic work was a collection of the times. Pre-dating this encyclopedia of knowledge was a babylonian clay tablet, Plimpton 322.

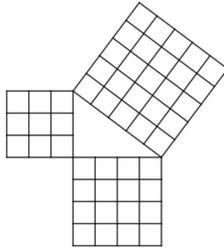
Example #1: What are some examples of Pythagorean triangles ?

|  |  |   |  |
|--|--|---|--|
| <p>Three sides are {3, 4, 5}</p> $a^2 + b^2 = c^2$                                 | <p>Three sides are {5, 12, 13}</p> $a^2 + b^2 = c^2$                               | <p>Three sides are {7, 24, 25}</p> $a^2 + b^2 = c^2$                                | <p>Three sides are {8, 15, 17}</p> $a^2 + b^2 = c^2$                                 |
|  |  |  |  |
| $9 + 16 = 25$ $25 = 25$  | $25 + 144 = 169$ $169 = 169$   | $49 + 576 = 625$ $625 = 625$  | $64 + 225 = 289$ $289 = 289$   |

Example #2: Which set of numbers could represent the side lengths of a right triangle ?

|   |   |   |
|---|---|---|
| <p>Three sides are {5, 10, 13}</p> $a^2 + b^2 = c^2$                                | <p>Three sides are {5, 12, 10}</p> $a^2 + b^2 = c^2$                                | <p>Three sides are {5, 12, 13}</p> $a^2 + b^2 = c^2$                                  |
|  |  |  |
| $25 + 100 \neq 169$ <p><b>NOT a right triangle</b></p>                              | $25 + 144 \neq 100$ <p><b>NOT a right triangle</b></p>                              | $25 + 144 = 169$ <p><b>A right triangle</b></p>                                       |

## Example #3: Area Representation



Notice also that this “symbol of the Pythagorean school” also represents with squares and a right triangle that the area of the base legs squared equals the square of the hypotenuse (the side opposite the right angle).

$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

$$25 = 25$$

Example: Plimpton 322

Row #5:

$$65^2 + 72^2 = 97^2$$

$$4225 + 5184 = 9409$$

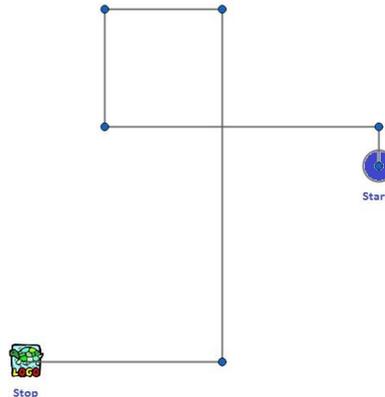
$$9409 = 9409$$

This clay tablet from the Babylonian Empire (300 BCE) contains a list of Pythagorean Triples in Sexagesimal (base 60) notation. The cuneiform wedge marks show a list that includes  $\{3,4,5\}$  dilated  $D_{15} = \{45,60,75\}$ . Also  $\{65,72,97\}$  and other integer values with decreasing slope ratios for each row representation. Was it for architectural use?

## Example: Mapping a journey

Seymour programs his robot “Kame” to move with a computer program. He instructs Kame to move with the following directions:

| Step | Direction     |
|------|---------------|
| 1    | 1 unit north  |
| 2    | 7 units west  |
| 3    | 3 units north |
| 4    | 3 units east  |
| 5    | 9 units south |
| 6    | 5 units west  |



The final displacement of the turtle is

9 units west, 5 units south.

Solution:

Make a right triangle with legs 9 and 5 units.

$$9^2 + 5^2 = d^2$$

$$81 + 25 = d^2$$

$$106 = d^2$$

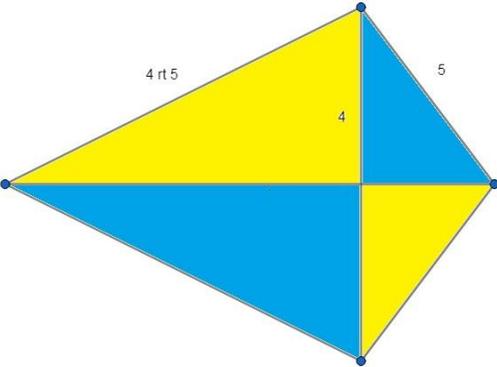
$$10.3 = d$$

The robot turtle is 10.3 units away from home.

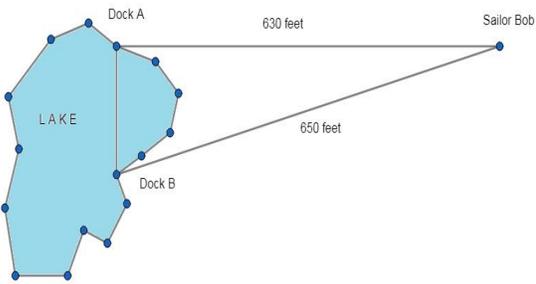
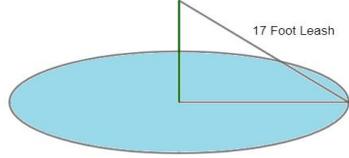
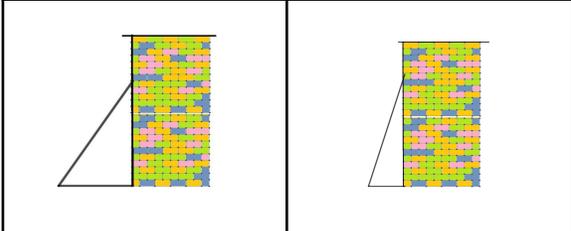
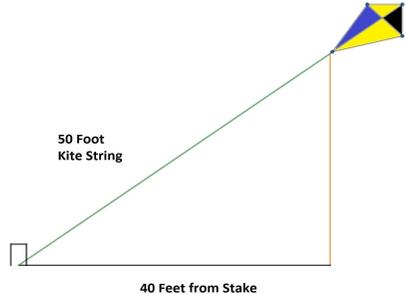
## SAT Lesson #17 Classwork: Pythagorean Theorem

|  |   |
|--|---|
| 1) What is the hypotenuse of a right triangle with legs $a = 5$ and $b = 12$ ?   | A) 9<br>B) 11<br>C) 13<br>D) 15   |
| 2) What is the length of the hypotenuse of a right triangle with legs $a = 2$ and $b = \sqrt{21}$ ?  | A) 4<br>B) 5<br>C) 6<br>D) 7  |
| 3) A 25 foot ladder leans against a building with the bottom of the ladder anchored 7 feet from the base. How far up the side of the building does the ladder reach? (Round your answer to the nearest tenth.)   | A) 18 feet<br>B) 20 feet<br>C) 22 feet<br>D) 24 feet                                    |
| 4) The plimpton 322 clay tablet at Columbia University contains an extraordinary pythagorean triplet. The hypotenuse is 18,541 and the length of the shorter leg is 12,709. Use a modern calculator to find the length of the other leg of the triangle. | A) 11,500<br>B) 12,500<br>C) 13,500<br>D) 14,500  |
| 5) A flagpole outside the school has a cable that a handler holds that is 37 feet long. The handler is standing 12 feet from the base of the pole. How tall is the flagpole?   | A) 33<br>B) 35<br>C) 37<br>D) 39  |
| 6) Which numbers do <i>not</i> represent the sides of a right triangle?  | A) $\{20, 21, 29\}$<br>B) $\{11, 60, 61\}$<br>C) $\{33, 56, 65\}$<br>D) $\{8, 14, 17\}$ |

## SAT Lesson #17: Classwork (Continued)

|   |   |
|---|---|
| 7) Which set of numbers could be the lengths of the sides of a right triangle ?   | A) {10, 24, 26}<br>B) {15, 22, 28}<br>C) {4, 8, 11}<br>D) {3, 4, 6}   |
| 8) Which set of numbers could NOT represent the side lengths of a right triangle ?  | A) {1, 4, $\sqrt{17}$ }<br>B) {2, 5, $\sqrt{29}$ }<br>C) {3, 7, $\sqrt{58}$ }<br>D) {3, 8, $\sqrt{77}$ }  |
| 9) Which set of numbers could NOT represent the side lengths of a right triangle ?  | A) { $\sqrt{3}$ , $\sqrt{13}$ , 4}<br>B) { $\sqrt{8}$ , $\sqrt{8}$ , 6}<br>C) { $\sqrt{11}$ , $\sqrt{38}$ , 7}<br>D) { $\sqrt{13}$ , $\sqrt{51}$ , 8} |
| 10) A radio tower that is 25 feet tall has 3 cables that keep the tower stable. Each cable is attached 5 feet below the top and is anchored 15 feet from the base of the tower. How much cable is supporting the tower ?  | A) 25<br>B) 50<br>C) 60<br>D) 75  |
| 11) A very tall 80 foot radio tower serves a large valley community. In order to keep the tower stable, 4 cables are attached every 25 feet and are anchored to the ground. The lower cables (at 25 feet) are 18.75 feet from the base. The middle cables (at 50 feet) are 37.5 feet from the base. The high cables (at 75 feet) are 100 feet from the base. What is the length of all 12 support cables together ? | A) 875<br>B) 900<br>C) 950<br>D) 1200   |
| 12) The kite below is designed with a cross brace that makes 4 right triangles. What is the length of the kite ?<br><i>measurements are { 4, 5 and <math>4\sqrt{5}</math> }</i><br><br>[ ----- Length ----- ]<br><br>  | A) 9<br>B) 10<br>C) 11<br>D) 12   |

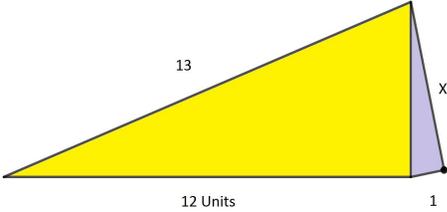
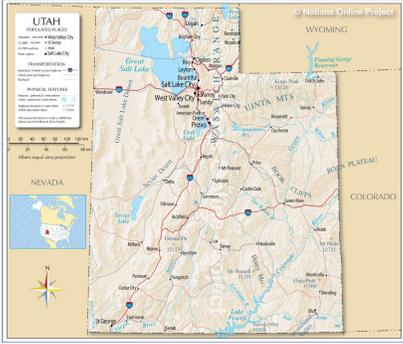
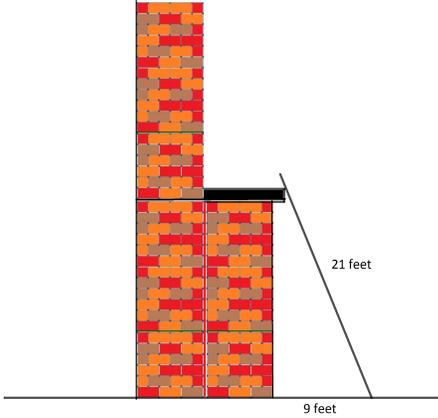
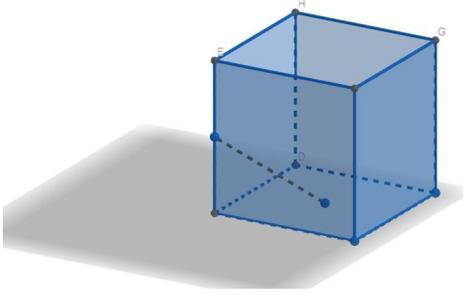
SAT Lesson #17: Classwork (Continued)

|   |  |  |
|---|--|--|
| <p>13) Sailor Bob would like to know how far it is by Boat across a lake, so he walks away from the charted course at Dock A for 630 feet at a right angle, then when he sees Dock B around the rocky coastline, he walks 650 feet to Dock B. Since a right triangle was formed, what is the Boat distance AB across the lake ?</p>                         |    | <p>A) 120 feet<br/>B) 150 feet<br/>C) 180 feet<br/>D) None of these.</p>                                       |
| <p>14) A dog is tied to a 5 foot pole at the top, and has a 17 foot leash. What is the square footage of the area the dog can roam ?</p>  | <p>A dog leash tied to a 5 foot pole.</p>    | <p>A) <math>264\pi</math><br/>B) <math>274\pi</math><br/>C) <math>294\pi</math><br/>D) <math>314\pi</math></p> |
| <p>15) A 12 foot ladder is leaning against a brick wall. The foot of the ladder is 6 feet from the base of the wall. If the ladder is moved 3 feet closer to the wall, how much higher will the top of the ladder touching the wall reach ?</p>   |  <p style="text-align: center;"><b>Ladder #1</b><br/>6 feet from wall</p> <p style="text-align: center;"><b>Ladder #2</b><br/>3 feet from wall</p> | <p>A) 2.4 feet<br/>B) 2.2 feet<br/>C) 1.2 feet<br/>D) 1.0 feet</p>   |
| <p>16) Mary and Betty-Ann decide to fly a kite together. Mary ties the kite to a stake they plant in a field, and her friend Betty-Ann walks 40 feet away from the stake and releases the kite, which rises over her head. It's very windy and the 50 foot string is under tension and straight as a bow string. How high is the kite above Betty-Ann ?</p> |  <p style="text-align: center;">50 Foot<br/>Kite String</p> <p style="text-align: center;">40 Feet from Stake</p>                                  | <p>A) 25 Feet<br/>B) 30 Feet<br/>C) 35 Feet<br/>D) 40 Feet</p>   |

## Chang Learning

## SAT Lesson 17: Pythagorean Theorem

## SAT Lesson #17: Classwork (Continued)

|   |  |   |
|---|--|---|
| <p>17) In the diagram at right, both triangles are right triangles. What is the value of <math>x</math> ?</p>   |    | <p>A) <math>\sqrt{22}</math><br/>           B) <math>\sqrt{24}</math><br/>           C) <math>\sqrt{30}</math><br/>           D) <math>\sqrt{32}</math></p>                   |
| <p>18) Map of Utah: The Western border of the state is 350 miles long. The southern border is 280 miles long. How long would a direct road from the Northwest corner to the Southeast corner of the state, to the nearest mile ?</p>                      |    | <p>A) 358 miles<br/>           B) 401 miles<br/>           C) 448 miles<br/>           D) 529 miles</p>   |
| <p>19) The Balcony: A worker has a 21 foot pole and leans the very top of it against a balcony to climb. With his construction tape measure he knows that the pole is 9 feet from the base of the wall. How high is the balcony to the nearest foot ?</p> |  | <p>A) 12 feet<br/>           B) 15 feet<br/>           C) 18 feet<br/>           D) 19 feet</p>   |
| <p>20) A cube of side length 5 is shown at right. What is the distance between the center of the base of the cube and the midpoint of the vertical edge, as shown ?</p>   |  | <p>A) <math>\frac{\sqrt{5}}{2}</math><br/>           B) <math>\frac{5\sqrt{3}}{2}</math><br/>           C) <math>\sqrt{6}</math><br/>           D) <math>5\sqrt{2}</math></p> |

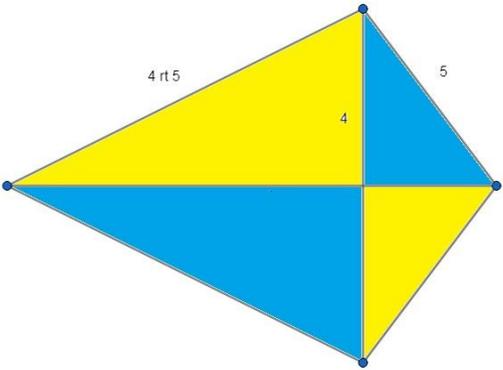
## SAT Lesson #17: Classwork SAT Exam Grid-In

|  |         |
|--|---------|
| <p><b>21. (Easy Level)</b></p> <p>Simplify: <math>a = 5, b = 2</math> (Use radians)</p> $\frac{a^2b + 3a - 2}{a + b} + (a - b + 1)^2 + a(\cos b\pi) + 1$                         | Grid-In |
| <p><b>22. (Easy Level)</b></p> <p>A worker earns \$15 per hour plus \$5 to \$25 per hour in tips. What is the least amount this worker is paid after working 30 hours ?</p>      | Grid-In |
| <p><b>23. (Mid Level)</b></p> <p>How many solutions are there to the system of equations:</p> $\begin{aligned} y &= 3x + 4 \\ -15x + 5y &= 5 \end{aligned}$                      | Grid-In |
| <p><b>24. (Mid Level)</b></p> <p>A NYC Restaurant offers 12 kinds of bagels. If five friends each choose a different type of bagel, how many different orders can be placed?</p> | Grid-In |
| <p><b>25. (Challenge Level)</b></p> <p>What are the product of the zeroes of the equation:</p> $h(t) = -9.8t^2 + 58.8t + 68.6$   | Grid-In |

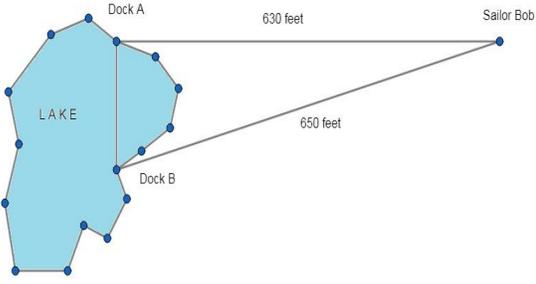
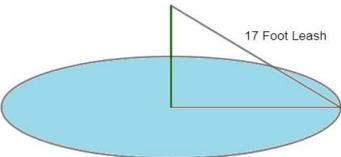
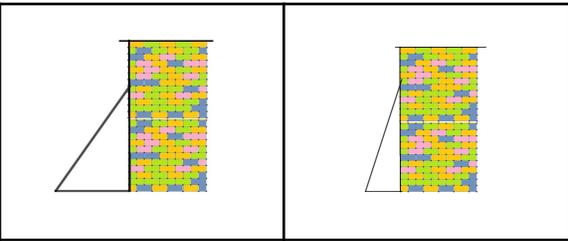
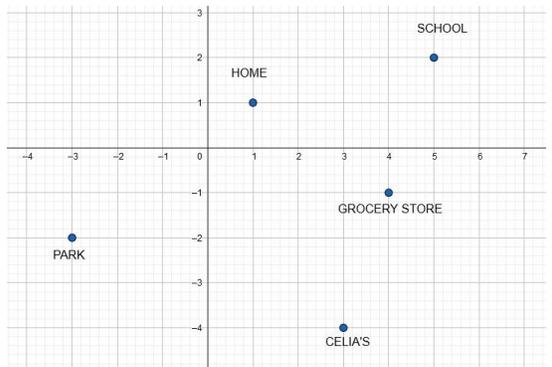
### SAT Lesson #17 Homework: Pythagorean Theorem

|   |   |
|---|---|
| <p>1) What is the hypotenuse of a right triangle with legs <math>a = 8</math> and <math>b = 15</math>?</p>  | <p>A) 12<br/>B) 17<br/>C) 19<br/>D) 23</p>  |
| <p>2) What is the length of the hypotenuse of a right triangle with legs <math>a = 9</math> and <math>b = \sqrt{19}</math>?</p>   | <p>A) 6<br/>B) 8<br/>C) 10<br/>D) 12</p>  |
| <p>3) A 18 foot ladder leans against a building with the bottom of the ladder anchored 6.5 feet from the base. How far up the side of the building does the ladder reach? (round to the nearest hundredth)</p>  | <p>A) 16.31 feet<br/>B) 16.43 feet<br/>C) 16.58 feet<br/>D) 16.79 feet</p>                    |
| <p>4) The plimpton 322 clay tablet at Columbia University contains an extraordinary pythagorean triplet. The hypotenuse is 8,161 and the length of the shorter leg is 4,961. Use a modern calculator to find the length of the other leg of the triangle.</p> | <p>A) 6400<br/>B) 6440<br/>C) 6480<br/>D) 6520</p>  |
| <p>5) A flagpole in central park stands tall at 45 feet. A park employee holds a rope from the flag atop that is held straight in tension and is 53 feet long. How far is the person standing from the base of the flagpole?</p>                              | <p>A) 14 feet<br/>B) 18 feet<br/>C) 20 feet<br/>D) 28 feet</p>                                |
| <p>6) Which numbers do <i>not</i> represent the sides of a right triangle?</p>  | <p>A) {56, 90, 106}<br/>B) {1771, 2700, 3229}<br/>C) {1679, 2400, 2929}<br/>D) {45,64,75}</p> |

## SAT Lesson #17: Homework (Continued)

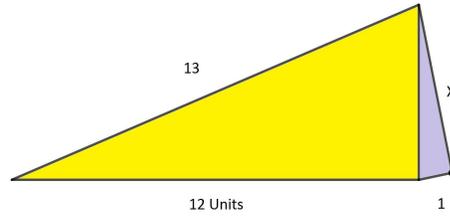
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|--|---|
| 7) Which set of numbers could be the lengths of the sides of a right triangle ?  | A) {12, 16, 20}<br>B) {18, 80, 81}<br>C) {7, 24, 26}<br>D) {8, 13, 17}  |
| 8) Which set of numbers could NOT represent the side lengths of a right triangle ?   | A) {2, 4, $2\sqrt{5}$ }<br>B) {3, 5, $\sqrt{34}$ }<br>C) {3, 9, $3\sqrt{10}$ }<br>D) {5, 7, $\sqrt{73}$ }   |
| 9) Which set of numbers could NOT represent the side lengths of a right triangle ?   | A) { $\sqrt{10}$ , $\sqrt{61}$ , 9}<br>B) { $\sqrt{65}$ , $\sqrt{35}$ , 10}<br>C) { $\sqrt{102}$ , $\sqrt{19}$ , 11}<br>D) { $\sqrt{110}$ , $\sqrt{34}$ , 12} |
| 10) A radio tower that is 25 feet tall has 3 cables that keep the tower stable. Each cable is attached 10 feet below the top and is anchored 20 feet from the base of the tower. How much cable is supporting the tower ?  | A) 60<br>B) 75<br>C) 90<br>D) 105   |
| 11) A very tall 80 foot radio tower serves a large valley community. In order to keep the tower stable, 3 cables are attached every 25 feet and are anchored to the ground. The lower cables (at 25 feet) are 18.75 feet from the base. The middle cables (at 50 feet) are 37.5 feet from the base. The high cables (at 75 feet) are 100 feet from the base. What is the length of all 9 support cables together ? | A) 616.25<br>B) 636.25<br>C) 656.25<br>D) 676.25  |
| 12) The kite below is designed with a cross brace that makes 4 right triangles. What is the area of the kite ? (The large and small triangles are congruent.)<br><br>[ ----- Length ----- ]<br><br>   | A) $12 + 16\sqrt{5}$<br>B) 44<br>C) 48<br>D) 52   |

**SAT Lesson #17: Homework (Continued)**

|   |   |  |
|---|---|--|
| <p>13) Sailor Bob would like to know how far it is by Boat across a lake, so he walks away from the charted course at Dock A a right angle 630 feet, then when he sees Dock B around the rocky coastline, he walks 650 feet to Dock B. He buys a commuter book with 20 one way tickets. What is the total distance of all the boat trips ?</p>  |   | <p>A) 3200 feet<br/>B) 3000 feet<br/>C) 2400 feet<br/>D) 2100 feet</p>   |
| <p>14) A cow is tied to a 5 foot pole on a grassy plain. At the top, and has a 17 foot leash. What is the square footage of the area the cow can graze ?</p>  | <p>A dog leash tied to a 5 foot pole.</p>   | <p>A) <math>264\pi</math><br/>B) <math>274\pi</math><br/>C) <math>294\pi</math><br/>D) <math>314\pi</math></p> |
| <p>15) An 18 foot ladder is leaning against a brick wall. The foot of the ladder is 6 feet from the base of the wall. If the ladder is moved 3 feet closer to the wall, how much higher will the top of the ladder touching the wall reach ? (round to the nearest tenth)</p>   |  <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <p><b>Ladder #1</b><br/>6 feet from wall</p> </div> <div style="text-align: center;"> <p><b>Ladder #2</b><br/>3 feet from wall</p> </div> </div> | <p>A) 0.4 feet<br/>B) 0.6 feet<br/>C) 0.8 feet<br/>D) 1.4 feet</p>   |
| <p>16) At right is a map of a local neighborhood. You can only walk on sidewalks along the straight lines shown. A student starts at home and goes to school, then after school walks with a friend, Celia with others to a park. Then they all go to a grocery store to shop. Then the friends all say goodbye at the store, and the student walks home. If the student took the direct route to each location, how many blocks total did the student walk in these commutes ?</p> |   | <p>A) 20 blocks<br/>B) 25 blocks<br/>C) 30 blocks<br/>D) 40 blocks</p>   |

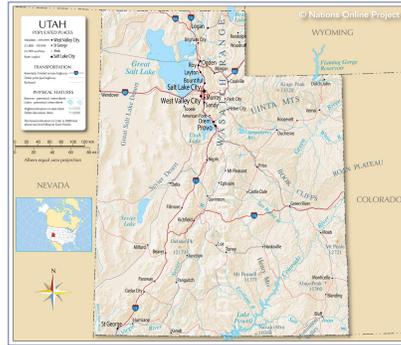
## SAT Lesson #17: Homework (Continued)

17) In the diagram at right, both triangles are right triangles. What is the total area of the figure ?



- A) 64.90  
B) 62.45  
C) 48.24  
D) 32.45

18) Map of Utah: The Western border of the state is 350 miles long. The southern border is 280 miles long. Approximately how much would it cost a driver with a vehicle that runs 35 mpg to take a trip on a direct road from the Northwest corner to the Southeast corner if gas is \$3.07 per gallon ?

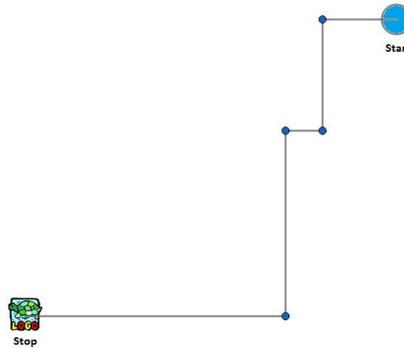


- A) \$39.32  
B) \$43.19  
C) \$45.55  
D) \$50.01

19) The robot turtle “Kame” is programmed to deliver a present along a route shown at right:

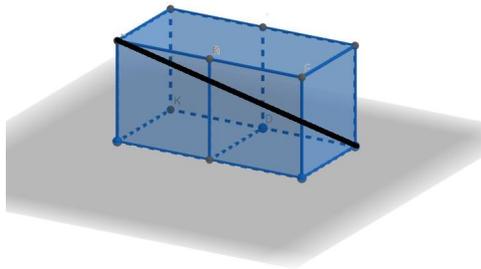
| Step | Command      |
|------|--------------|
| 1    | 2 units left |
| 2    | 3 units down |
| 3    | 1 unit left  |
| 4    | 5 units down |
| 5    | 7 units left |

How far away is Kame, start to finish ?



- A) 11.6 feet  
B) 12.8 feet  
C) 13.4 feet  
D) 13.9 feet

20) A double cube is constructed by placing adjacent cubes of side length 5 as shown at right. What is the distance of the long diagonal, as shown ?



- A)  $6\sqrt{5}$   
B)  $\frac{10\sqrt{5}}{2}$   
C)  $\frac{5\sqrt{3}}{2}$   
D)  $5\sqrt{6}$

SAT Lesson #17: Homework SAT Exam Grid-In

*Homework #21 This problem is taken from a Chinese mathematics textbook called Chui-chang suan-shu, or Nine Chapters on the Mathematical Art, which was written about 250 B.C. A 10-ft-long stem of bamboo is broken in such a way that its tip touches the ground  $t = 3$  ft from the base of the stem, as shown in the figure. What is the height of the break? (Hint: Use the Pythagorean Theorem.)*



## SAT Lesson #17: Homework SAT Exam Grid-In

|  |                |
|--|----------------|
| <p><b>22. (Easy Level)</b></p> <p>Simplify: <math>a = 5, b = 2</math> (Use radians)</p> $\frac{a^2b + 3a - 2}{a + b} + (a - b + 1)^2 + a(\sin b\pi) + 1$                               | <b>Grid-In</b> |
| <p><b>23. (Mid Level)</b></p> <p>A worker earns \$16 per hour plus \$10 to \$30 per hour in tips. What is the least amount this worker is paid after working 30 hours ?</p>            | <b>Grid-In</b> |
| <p><b>24. (Mid Level)</b></p> <p>How many solutions are there to the system of equations:</p> $y = 3x + 7$ $- 15x + 5y = 19$   | <b>Grid-In</b> |
| <p><b>25. (Challenge Level)</b></p> <p>A NYC Restaurant offers 12 kinds of bagels. If four friends each choose a different type of bagel, how many different orders can be placed?</p> | <b>Grid-In</b> |