Chang Learning Center

SAT: Studying for the SAT Mathematics Section

Lesson #6: Ratios and Proportions Part 1

July 17th, 2024

By Joshua Weiner

Provided by Chang Learning

SAT Quiz #5
Review
Questions {#4 and #5}

4) (Mid Level)

Let $f(x) = 3x + 5$ and $g(x) = \frac{1}{3}x^2 + 2x + 2$.

What is the value of $g(f(\frac{-8}{3}))$?

A) -1

B) 17

C) $\frac{-26}{27}$

D) $\frac{19}{9}$

5) **(Challenge Level)** If the shortest distance between $(x, y) = (3, 4)$ and $(x, y) = (-5, 12)$ on the Cartesian plane is measured as $a\sqrt{b}$ in simplest form, what is $a + b$?	Grid In

SAT Homework #5

Review

Questions {#2, #4, and #19}

Question 1

1 pts

If $f(x) = 5x - 1$, what is the value of $f(2)$?

- (A) -1
- (B) 2
- (C) 5
- (D) 9

Question 2

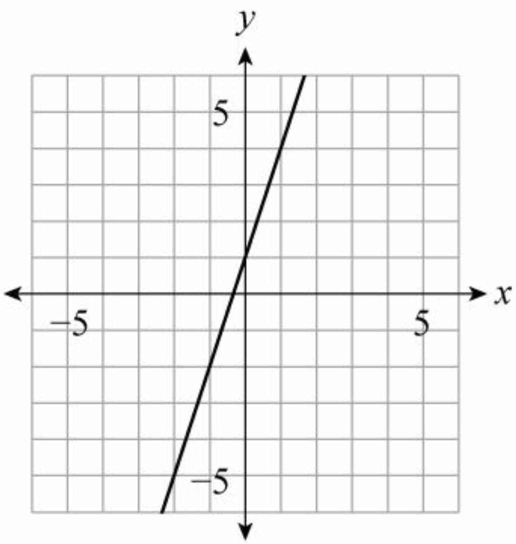
1 pts

A computer program takes as input the number of students in a class with brown hair and returns as output the predicted number of students in the class with blue eyes. Which of the following could be a valid pair of inputs and outputs for this computer program?

- (A) $(5, 18)$
- (B) $(3, -8)$
- (C) $(20, 5.5)$
- (D) $(21.5, 0)$

Question 3

1 pts



The graph of $y = f(x)$ is shown. What is the value of $f(1)$?

Question 4

1 pts

If $f(x) = 2x + 7$, what is the value of x when $f(x) = 35$?

- (A) $\frac{1}{7}$
- (B) 7
- (C) 14
- (D) 77

Question 5

1 pts

x	$f(x)$
-1	1
0	-1
1	-3

Which of the following best models the data for the linear function given in the table?

- (A) $f(x) = -2x - 2$
- (B) $f(x) = -2x - 1$
- (C) $f(x) = -x - 2$
- (D) $f(x) = -x - 1$

Question 6

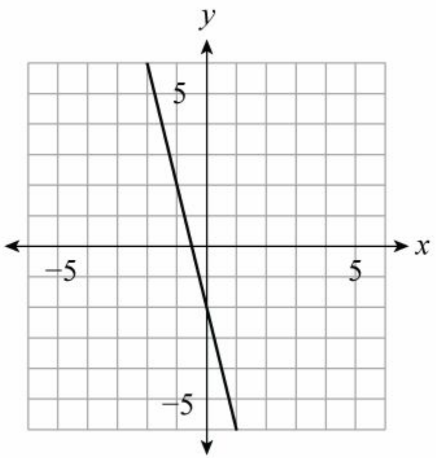
1 pts

$$f(x) = \frac{2}{3}x - 1$$
$$g(x) = 5x + 3$$

The functions f and g are given. What is the value of $g(f(6))$?

- (A) 3
- (B) 18
- (C) 21
- (D) 99

Question 7 1 pts



The function $f(x)$ is shown in the graph. If $g(x) = (\frac{1}{2})f(x) + 3$, which of the following could be the equation of $g(x)$?

- (A) $f(x) = -4x - 2$
- (B) $f(x) = -2x - 1$
- (C) $f(x) = -2x + 2$
- (D) $f(x) = -x + 4$

Question 8

1 pts

In the linear function $f(x)$, $f(2m) = 0$ and $2f(0) = m$. What is the slope of $f(x)$?

Question 9

1 pts

x	$f(x)$
$-a$	$8a$
0	0
$2a$	$-16a$

The table describes a linear equation. If $f(a) = 32$, what is the value of a?

- (A) -4
- (B) -2
- (C) 2
- (D) 4

Question 10

1 pts

If $f(x) = -x + 1$, for what value of x is $f(5x) = f(x+5)$?

- (A) -5
- (B) $\frac{4}{5}$
- (C) $\frac{5}{4}$
- (D) 5

SAT Lesson #6
Problem Solving & Data Analysis
Ratios & Proportions Part 1

[PART 2B]

PROBLEM-SOLVING AND DATA ANALYSIS

[CHAPTER 8]

RATES, RATIOS, PROPORTIONS, PERCENTS, AND UNITS

LEARNING OBJECTIVES

After completing this chapter, you will be able to:

- Given any two values in a three-part rate equation, solve for the third
- Set up and solve a proportion for a missing value
- Use ratios to perform unit conversions
- Calculate percents
- Calculate percent change

Math

Rates

LEARNING OBJECTIVE

After this lesson, you will be able to:

- Given any two values in a three-part rate equation, solve for the third

To answer a question like this:

Tanks A and B are both used to store the same chemical. Tank A, which holds 108 gallons when full, takes 18 minutes to empty when the valve is opened. Tank B, which holds 196 gallons when full, takes 28 minutes to empty when the valve is opened. What is the positive difference, in gallons per minute, between the rates at which tanks A and B empty?

A) 1

B) 6

C) 7

D) 9

You need to know this:

A **rate** is an expression of the amount of something done per unit of time. Speed, for example, represents the distance something travels in a particular amount of time. The general three-part rate formula is **Rate** $= \frac{\textbf{amount}}{\textbf{time}}$, although you may see this in a number of different ways. With a little math, the equation becomes **Amount = rate × time**, or you might find it helpful to use **Distance = rate × time** or even **Work = rate × time**. We'll use several ways in this chapter. For example, say a cyclist rides 30 miles in 2 hours. To determine her speed(rate), plug in 30 miles for the amount and 2 hours for the time to get Speed $= \frac{30 \text{ miles}}{2 \text{ hours}} =$ 15 miles per hour. Since a rate also represents the work done in one unit of time, its reciprocal represents the amount of time taken to do one unit of work. The bike being ridden at 15 miles per hour takes $\frac{1}{15}$ of an hour to ride one mile.

Like other three-part formulas, you may need to rearrange the rate formula to solve for the other two parts: the previously mentioned **Amount = rate × time** as well as **Time** $= \frac{\textbf{amount}}{\textbf{rate}}$.

Sometimes, you'll be given the rates of two separate workers attempting to complete a task, and you'll need to combine their rates to figure out how long they'd take to complete the task together. Suppose that it takes Dorian 3 hours to paint a room and Clarice 2 hours to paint a room of the same size. So, Dorian's rate is $\frac{1 \text{ room}}{3 \text{ hours}} = \frac{1}{3}$ room per hour and Clarice's rate is $\frac{1 \text{ room}}{2 \text{ hours}} = \frac{1}{2}$ room per hour. If they work together, their combined progress is the sum of their individual rates: $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$ room per hour. You could then take the reciprocal to find that it would take them $\frac{6}{5}$ of an hour to paint one room.

There is also a simplified formula for two workers. If a and b represent the time each worker takes to complete the task and T is their combined time, then $T = \frac{ab}{a+b}$.

You need to do this:

- Determine which two parts of the rate formula are given and use them to solve for the third.
- If there are multiple workers collaborating on a task, use their individual rates to determine their combined rate.

Explanation:

First, find the rates at which each individual tank drains using the rate formula, **Rate** $= \frac{\textbf{amount}}{\textbf{time}}$. For tank A, the amount is 108 gallons and the time is 18 minutes, so Rate $= \frac{108 \text{ gallons}}{18 \text{ minutes}} = 6$ gallons per minute. For tank B, the amount is 196 gallons and the time is 28 minutes, so Rate $= \frac{196 \text{ gallons}}{28 \text{ minutes}} = 7$ gallons per minute. Remember, though, that the question asks for the positive difference between these rates, not either of the rates themselves. Since $7 - 6 = 1$, **(A)** is correct.

Ratios and Proportions

LEARNING OBJECTIVE

After this lesson, you will be able to:

- Set up and solve a proportion for a missing value

To answer a question like this:

The aircraft carrier *Essex* was 872 feet long with a beam (width) of 147 feet. A museum wishes to build an exact replica scale model of the *Essex* that is 8 feet long. Approximately how many feet wide will the scale model's beam be?

A) 1.35

B) 2.14

C) 2.68

D) 5.93

Page 122

You need to know this:

A **ratio** is a comparison of one quantity to another. When writing ratios, you can compare one part of a group to another part of that group, or you can compare a part of the group to the whole group. Suppose you have a bowl of apples and oranges: you can write ratios that compare apples to oranges (part to part), apples to total fruit (part to whole), and oranges to total fruit (part to whole).

Keep in mind that ratios convey *relative* amounts, not necessarily actual amounts, and that they are typically expressed in lowest terms. For example, if there are 10 apples and 6 oranges in a bowl, the ratio of apples to oranges would likely be expressed as $\frac{5}{3}$ on the SAT rather than as $\frac{10}{6}$. However, if you know the ratio of apples to oranges and either the actual number of apples or the total number of pieces of fruit, you can find the actual number of oranges by setting up a proportion.

Note that the SAT may occasionally use the word "proportion" to mean "ratio."

A **proportion** is simply two ratios set equal to each other, such as $\frac{a}{b} = \frac{c}{d}$. Proportions are an efficient way to solve certain problems, but you must exercise caution when setting them up. Noting the units of each piece of the proportion will help you put each piece of the proportion in the right place.

Sometimes the SAT may ask you to determine whether certain proportions are equivalent—check this by cross-multiplying. You'll get results that are much easier to compare.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then: } ad = bc, \frac{a}{c} = \frac{b}{d}, \frac{d}{b} = \frac{c}{a}, \frac{b}{a} = \frac{d}{c}, \text{ BUT } \frac{a}{d} \neq \frac{c}{b}$$

Each derived ratio shown except the last one is simply a manipulation of the first, so all except the last are correct. You can verify this via cross-multiplication ($ad = bc$ in each case except the last).

Alternatively, you can pick equivalent fractions $\frac{2}{3}$ and $\frac{6}{9}$ ($a = 2$, $b = 3$, $c = 6$, $d = 9$). Cross-multiplication gives $2 \times 9 = 3 \times 6$, which is a true statement. Dividing 2 and 3 by 6 and 9 gives $\frac{2}{6} = \frac{3}{9}$, which is also true, and so on. However, attempting to equate $\frac{2}{9}$ and $\frac{3}{6}$ will not work.

If you know any three numerical values in a proportion, you can solve for the fourth. For example, say a fruit stand sells 3 peaches for every 5 apricots, and you are supposed to calculate the number of peaches sold on a day when 20 apricots were sold. You would use the given information to set up a proportion and solve for the unknown:

$$\frac{3}{5} = \frac{p}{20}$$

You can now solve for the number of peaches sold, p, by cross-multiplying:

$$60 = 5p$$
$$p = 12$$

Alternatively, you could use the common multiplier to solve for p: the numerator and denominator in the original ratio must be multiplied by the same value to arrive at their respective terms in the new ratio. To get from 5 to 20 in the denominator, you multiply by 4, so you also have to multiply the 3 in the numerator by 4 to arrive at the actual number of peaches sold: $4(3) = 12$.

You need to do this:

Set up a proportion and solve for the unknown, either by cross-multiplying or by using the common multiplier.

Explanation:

The ratio of the length of the real *Essex* to that of the scale model is $\frac{872 \text{ ft}}{8 \text{ ft}}$. You know the actual beam width (147 feet), so set up a proportion and solve for the scale model's beam width:

$$\frac{872 \text{ ft}}{8 \text{ ft}} = \frac{147 \text{ ft}}{x \text{ ft}}$$
$$872x = 1{,}176 \text{ ft}$$
$$x \approx 1.349 \text{ ft}$$

The correct answer is **(B)**.

SAT Lesson #6
Problem Solving and Data Analysis
Classwork (15 Questions)

Pages 120-121 Questions #1 to #8
Pages 124-125 Questions #9 to #15

1

A sprinter can run 29 feet in 1 second. How many seconds will it take the sprinter to run 300 feet?

______

2

Pump A can transfer 3 gallons of water per minute, and pump B can work twice as fast as pump A. How many minutes will it take pump A and pump B, working together, to transfer 279 gallons of water?

Ⓐ 30

Ⓑ 31

Ⓒ 46.5

Ⓓ 93

3

An airplane travels 900 miles at 300 miles per hour and then an additional 500 miles at 250 miles per hour. What is the airplane's average speed, in miles per hour, over the entire trip?

Ⓐ 260

Ⓑ 275

Ⓒ 280

Ⓓ 290

4

An asteroid travels 30,000 kilometers per hour. It rotates on its axis one time for every 600,000 kilometers it travels. How many hours will it take for the asteroid to rotate on its axis 12 times?

Ⓐ 20

Ⓑ 24

Ⓒ 120

Ⓓ 240

5

HINT: For Q5, calculate the distance left to travel in the time remaining.

At noon, a car departs on a 270-mile trip. It travels for the first 1.5 hours at 40 miles per hour. It completes the trip at 5:00 p.m. on the same day. What was the car's average speed, in miles per hour, from 1:30 to 5:00 p.m.?

Ⓐ 35

Ⓑ 40

Ⓒ 60

Ⓓ 70

6

Tavish can clean his room in 2 hours. His brother can clean the same room in 4 hours. If they work together, how many hours will it take them to clean their room?

Ⓐ 1

Ⓑ $\frac{4}{3}$

Ⓒ $\frac{3}{2}$

Ⓓ 6

7

HINT: For Q7, determine the net rate at which the pond is being filled.

A 2,100 gallon fish pond has a circulation drain at the bottom that drains at a rate of 20 gallons per hour. A pump adds water to the pond at a rate of 55 gallons per hour. If the pond starts out empty, how many hours will it take the pump to completely fill it?

______

8

If cyclist A travels at 6 miles per hour and cyclist B travels at 8 miles per hour, how much longer, in hours, will it take cyclist A than cyclist B to travel 24 miles?

Ⓐ $\frac{3}{8}$

Ⓑ $\frac{3}{4}$

Ⓒ 1

Ⓓ 2

9

The number of topics teachers at a certain school can cover is directly proportional to the length of time they have to review. If teachers can cover 9 topics in a single 45-minute period, how many topics can they cover in a 60-minute period?

______

10

One pound on Earth is equal to approximately 0.166 pounds on the Moon. If a person weighs 29 pounds on the Moon, approximately how much, in pounds, does the person weigh on Earth?

Ⓐ 21

Ⓑ 48

Ⓒ 175

Ⓓ 196

11

A machine produces 6 defective parts out of every 3,500 it makes. How many total parts were made during the time the machine produced 27 defective parts?

Ⓐ 14,000

Ⓑ 15,750

Ⓒ 17,500

Ⓓ 21,000

12

HINT: For Q12, designate the unknown starting number of first-year students and second-year students with the ratio $\frac{f}{s} = \frac{3}{10}$.

The ratio of first-year students to second-year students in an auditorium was 3 to 10. After an additional 270 first-year students and 120 second-year students entered the auditorium, the ratio of first-year students to second-year students was 6 to 5. No other students entered or left the auditorium. How many first-year students were in the auditorium before the additional students entered?

13

Riding her bicycle, Reyna can travel 1 mile in 5.5 minutes. If she rides at a constant rate, which of the following is closest to the distance she will travel in 90 minutes?

Ⓐ 9 miles

Ⓑ 11 miles

Ⓒ 13 miles

Ⓓ 16 miles

14

If $\frac{x+y}{x} = \frac{4}{9}$, which of the following proportions is equivalent?

Ⓐ $\frac{y}{x} = -\frac{5}{9}$

Ⓑ $\frac{y}{x} = \frac{13}{9}$

Ⓒ $\frac{y-x}{x} = -\frac{4}{9}$

Ⓓ $\frac{y-x}{x} = -\frac{9}{4}$

15

HINT: For Q15, start with the proportion $\frac{\text{physicists}}{\text{total}} = \frac{2}{5}$, then think about what to substitute for "physicists" and "total."

All of the attendees at a symposium are either physicists or biologists. If there are 123 physicists and 270 biologists, then how many additional physicists must arrive at the symposium in order for the ratio of physicists to total attendees to become 2 to 5?

Ⓐ 25

Ⓑ 50

Ⓒ 57

Ⓓ 114

SAT Classwork #6: Problem Solving and Data Analysis

Rates
Pages 120-121

1) 10.34	5) C
2) B	6) B
3) C	7) 60
4) D	8) C

Ratios and Proportions
Pages 124-125

9) 12	13) D
10) C	14) A
11) B	15) C
12) 42	

SAT Section 2 - Math Module 1

No calculators allowed

43 minutes to complete 27 questions

SAT Section 2 - Math Module 2

Use your calculator

43 minutes to complete 27 questions

A few Test-Taking Strategies

- Prepare in an organized way: Focus on ALGEBRA, GEOMETRY, COORDINATE PLANE, CHARTS & GRAPHS and STATISTICS lessons from Grades 9-10

- Be comfortable with the SAT Level of questions by exposure to as many practice questions as possible. The SAT is a patterned exam.

- Work on Time Management. Be sure to complete "easy to mid" level questions first.

- Some multiple choice questions can be solved by PLUG IN of the answer choices.
- Some multiple choice questions can be simplified by PLUG IN A VALUE for the variable (Plug in "1,2,3,4 or 5")
- ESTIMATE the answer to save procedural time on questions.

- Study and MEMORIZE FORMULAS and SOLUTION METHODS before the exam.

- Look for SHORTCUTS

Chang Learning Center
SAT Preparation

Mathematics

Quiz
Lesson
Homework