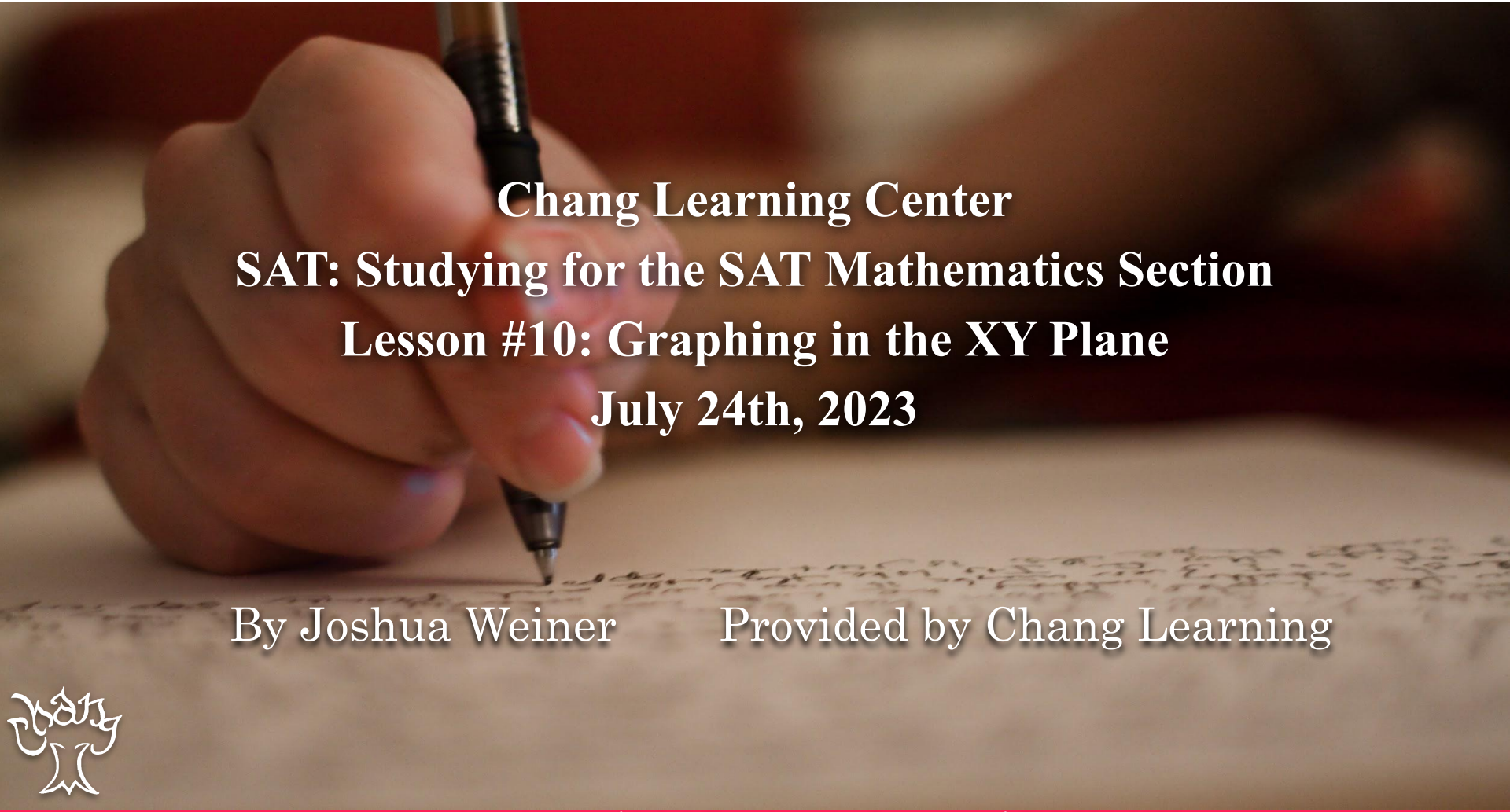


Chang Learning Center
SAT: Studying for the SAT Mathematics Section
Lesson #10: Graphing in the XY Plane
July 24th, 2023

By Joshua Weiner Provided by Chang Learning

SAT Quiz #9
Review
Questions {#3 and #5}

1) **(Easy Level)**

If $24x - 6 = 30$, what is the value of $4x + 5$?

A) 4
B) 15
C) 7
D) 11

2) (Easy Level)

A winter flu medicine costs $18 per bottle. The school principal buys a case of 24 bottles for the entire school population. Each bottle has enough for 40 prescriptions. What is the unit cost for three doses of flu medicine ?

A) $2.25
B) $1.80
C) $1.35
D) $0.90

3) (Mid Level)

Which of the following is the equation of a circle with center (2,0) and with point $(5, \sqrt{7})$ on the circumference of the circle ?

A) $(x-2)^2 + y^2 = 4$

B) $(x-2)^2 + y^2 = 16$

C) $(x-25)^2 + (y-7)^2 = 4$

D) $(x+2)^2 + y^2 = 16$

4) (Mid Level)

The Patas Monkey can live up to 20 years on average in the wild in Africa. A scientist is following an adult herd of Patas Monkeys with the ages {18, 21, 16, 14, 11, 16, 17, 15} years. What is the average age of the herd ?

A) 16
B) 15
C) 14
D) 13

5) (Challenge Level)

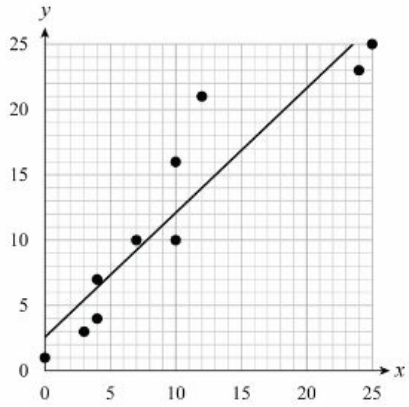
Grid In

Carol saves $1000 which is earning 6% quarterly compounded interest each year. How much will the savings be worth after 20 years ? (Rounded to the nearest $10 dollars)

SAT Homework #9
Review
Questions {#, #, #}

Question 1

1 pts



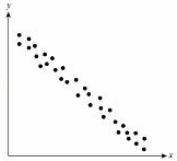
The given scatterplot shows the relationship between two variables, x and y, as well as a line of best fit for the data. At $x = 10$, which of the following is closest to the y-value predicted by the line of best fit?

- (A) 8
- (B) 10
- (C) 12
- (D) 16

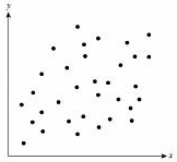
Question 2 1 pts

Which of the following graphs most clearly shows a strong positive association between x and y?

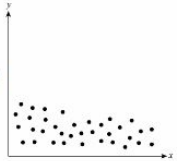
(A)



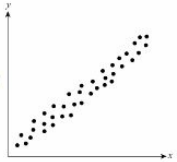
(B)



(C)

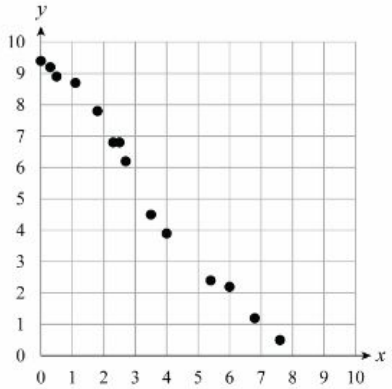


(D)



Question 3

1 pts

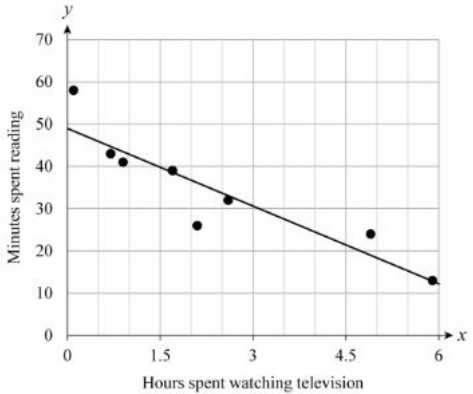


Which of the following equations could be an appropriate model for the data shown in the scatterplot?

- (A) $y = -1.26x + 9.6$
- (B) $y = -1.26x - 9.6$
- (C) $y = 1.26x + 9.6$
- (D) $y = 1.26x - 9.6$

Question 4

1 pts

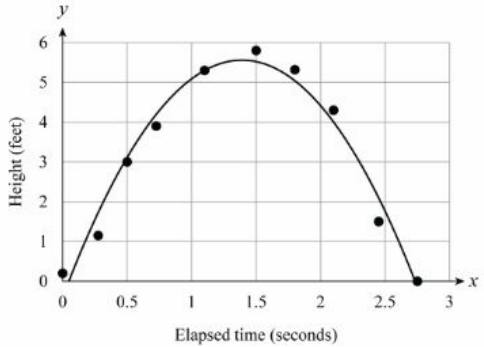


The scatterplot shows the relationship between the number of hours a fifth grader spent watching television per day and the number of minutes the same 5th grader spent reading per day, for eight different days. A line of best fit is also shown. Based on this data, which of the following is closest to the number of minutes that the fifth grader would be expected to spend reading on a day that she also spends 4 hours watching television?

- (A) 7
- (B) 21
- (C) 25
- (D) 31

Question 5

1 pts

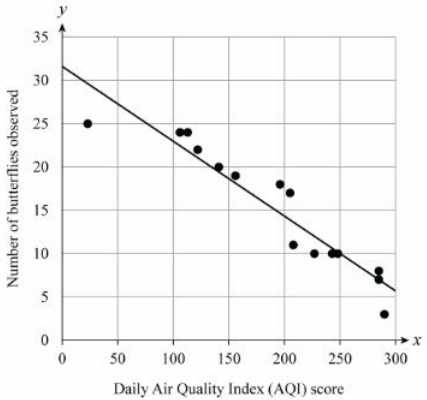


An air pump is used to launch a soft foam rocket. The scatterplot shows the relationship between the elapsed time, in seconds, since the rocket was launched and the rocket's height, in feet. A curve of best fit is also shown. Which of the following could be the equation for the curve of best fit?

- (A) $y = -3x^2 + 8.5x - 0.4$
- (B) $y = 3x^2 + 8.5x - 0.4$
- (C) $y = -5x^2 - 6x + 1.5$
- (D) $y = 5x^2 - 6x + 1.5$

Question 6

1 pts

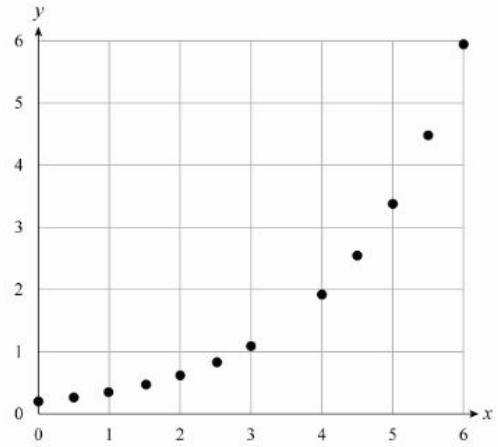


A scientist recorded the number of butterflies observed in a public park on each of fifteen different days. The graph shows the number of butterflies observed each day plotted against the park's daily score on the Air Quality Index (AQI), a system used to estimate the amount of pollution present in the air. A line of best fit for the data is also shown. According to the graph, which of the following statements is true about the relationship between the number of butterflies observed in the park and the park's daily AQI score?

- (A) There is no association between the number of butterflies observed in the park and the park's daily AQI score.
- (B) Fewer butterflies tend to be observed as the park's daily AQI score approaches 150, but the number of butterflies observed stays about the same as the daily AQI score continues to increase.
- (C) Fewer butterflies tend to be observed in the park on days with lower AQI scores.
- (D) More butterflies tend to be observed in the park on days with lower AQI scores.

Question 7

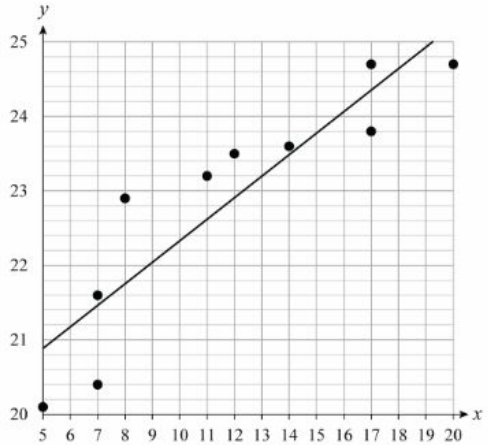
1 pts



Which of the following equations is the most appropriate model for the data shown in the scatterplot?

- (A) $y = 0.2(1 - 0.76)^x$
- (B) $y = 0.2(1 + 0.76)^x$
- (C) $y = 0.2(x^2 - 0.76)$
- (D) $y = 0.2(x^2 + 0.76)$

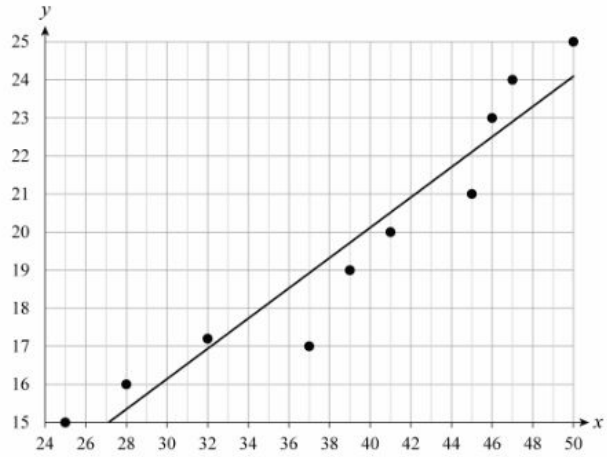
Question 8 1 pts



The given scatterplot shows the relationship between two variables, x and y. A line of best fit for the data is also shown. What is the positive difference, rounded to the nearest tenth, between the expected value of y and the actual value of y when $x = 11$?

Question 9

1 pts

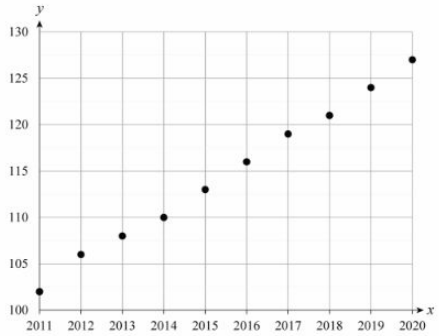


The given scatterplot shows the relationship between two variables, x and y, as well as a line of best fit for the data. Which of the following is closest to the y-value predicted by the line of best fit when $x = 60$?

- (A) 16.7
- (B) 22.1
- (C) 28.2
- (D) 43.4

Question 10 1 pts

Region	Growth Factor
North	1.8
East	2.7
South	4.2
West	5.0



A population study was conducted in a certain country. The data was used to calculate an average growth factor for each of the country's four regions, and the average growth factor for each region is reported in the table. The scatterplot shows the population data, in thousands, from one of the four regions that were collected during the study. Which region's population data is shown in the scatterplot?

- (A) North
- (B) East
- (C) South
- (D) West

SAT Lesson #10

Absolute Value & Non-Linear Equations

[PART 2C]

ADVANCED MATH

[CHAPTER 11]

ABSOLUTE VALUE AND NONLINEAR FUNCTIONS

LEARNING OBJECTIVES

After completing this chapter, you will be able to:

- Solve an equation containing an absolute value expression
- Interpret the graph of an equation containing an absolute value expression
- Interpret the domain, range, and properties of nonlinear functions and their graphs
- Evaluate the output of a given nonlinear function

Absolute Value

LEARNING OBJECTIVES

After this lesson, you will be able to:

- Solve an equation containing an absolute value expression
- Interpret the graph of an equation containing an absolute value expression

To answer a question like this:

If $|2x - 14| = 4$ and $x^2 + 11 = 36$, what is the value of x?

A) -5

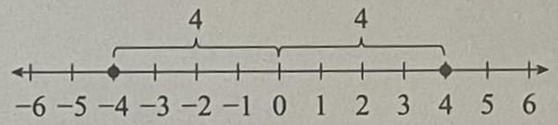
B) 5

C) 6

D) 9

You need to know this:

The **absolute value** of a number represents its distance from zero on a number line. It is represented by a set of vertical lines around a number, variable, or expression. For example, $|x| = 4$ means that x is four units away from zero on the number line, which means that x itself could be either 4 or -4.

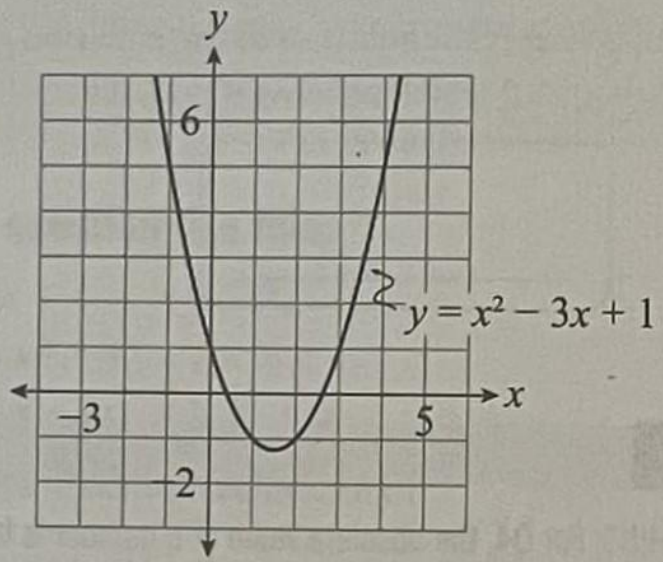


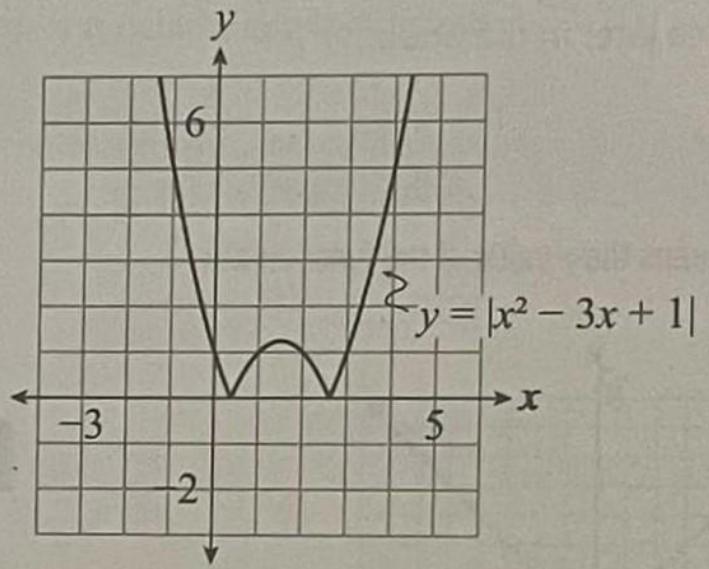
Thus, the effect of putting something inside a set of absolute value bars is to take any nonzero number and make it positive (if it wasn't already). The absolute value of 0 is simply 0.

When you see an equation involving absolute value, you'll need to consider both possible values of the expression inside the absolute value bars. Say that $|x + 8| = 11$. Then, you know that either $x + 8 = 11$ or $x + 8 = -11$, and you could solve each equation to find that $x = 3$ or $x = -19$. Both of these possible values for x satisfy the original equation; if a question asks you to narrow it down to a single value for x, you'd need more information—such as another equation.

You may see a question involving an absolute value function, where the entire function is in absolute value bars. The domain of such a function is the same as it would be if there were no absolute value bars. For instance, $f(x) = |x^2 - 3|$ has a domain of all real numbers, since any real number can be squared, while $f(x) = \left|\frac{1}{x+1}\right|$ has a domain of all real numbers other than -1, since $\frac{1}{x+1}$ is undefined when the denominator is 0. The range of an absolute value function will consist only of non-negative numbers because the absolute value bars turn any otherwise negative output positive.

The graph of an absolute value function will look similar to what it would look like without the absolute value bars; the only difference is that any point on the graph that would have appeared below the y-axis will instead appear an equal distance above the x-axis. For instance, consider the graphs of $f(x) = x^2 - 3x + 1$ and $f(x) = |x^2 - 3x + 1|$:





You need to do this:

- Set up equations with both the positive and negative values of the expression inside the absolute value bars.
- If needed, use any other given information to narrow down the possible values of x.
- In some more complicated absolute value equations, you may need to check for extraneous solutions by plugging in the potential solutions you've found back into the original equation to see if they work.

Explanation:

You need a value of x that satisfies both of these equations. Start by considering the absolute value equation. If $|2x - 14| = 4$, then $2x - 14 = 4$ or $2x - 14 = -4$. Solve each of these equations: $2x = 18$ and $x = 9$, or $2x = 10$ and $x = 5$. Now, solve the other equation. If $x^2 + 11 = 36$, then $x^2 = 25$, and $x = \pm 5$. Thus, $x = 5$ is the only solution that satisfies both equations, and **(B)** is correct.

Nonlinear Functions

LEARNING OBJECTIVES

After this lesson, you will be able to:

- Interpret the domain, range, and properties of nonlinear functions and their graphs
- Evaluate the output of a given nonlinear function

To answer a question like this:

$$f(x) = \begin{cases} 2x + 4, & x \leq -2 \\ -x^2 + 4, & -2 < x < 3 \\ -x - 2, & x \geq 3 \end{cases}$$

What is the maximum value of $f(x)$?

A) 0

B) 2

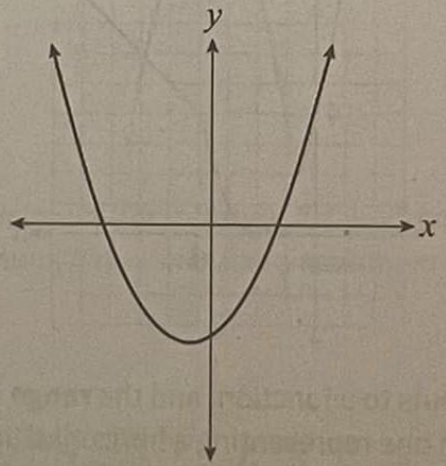
C) 4

D) $f(x)$ does not have a maximum.

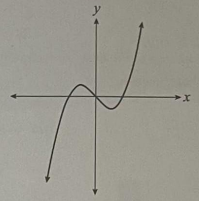
You need to know this:

You learned about linear functions earlier in the book. Like a linear function, a nonlinear function takes a number as input and generates a unique output. Unlike a linear function, however, a nonlinear function's graph will be something other than a line. Here are some examples of nonlinear functions you may encounter on the SAT:

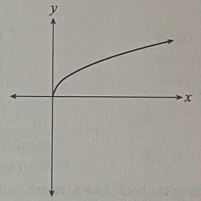
- Parabola: A curve generated by a function in the form $f(x) = ax^2 + bx + c$. You'll learn more about parabolas in the chapter on quadratics.



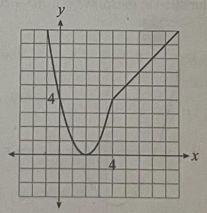
- Cubic function: A curve generated by a function with an x^3 term. The function's graph can cross the x-axis either one, two, or three times.



- Square root function: A curve generated by a function with a $\sqrt{x}$ term.

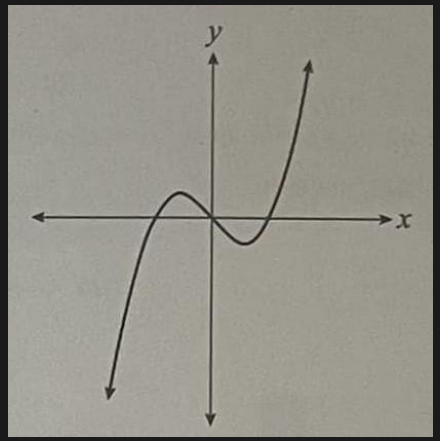


- Piecewise function: A function may have different equations for different x values. For instance, a function may be defined as $f(x) = \begin{cases} (x-2)^2, x \leq 4 \\ x, x > 4 \end{cases}$

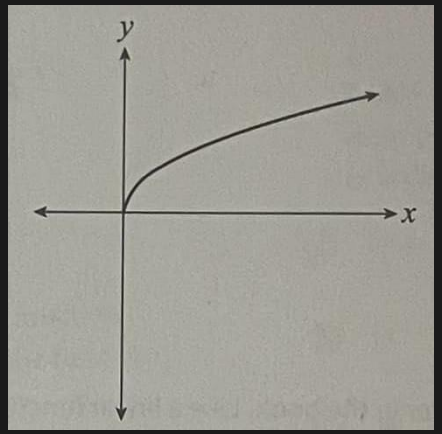


The **domain** is the set of all possible inputs to a function, and the **range** is the set of all possible outputs of a function. In a linear function, other than one representing a horizontal line, the domain and range both consist of all real numbers; a line is infinitely long, and every possible x value has a different associated y value. In a nonlinear function, you'll have to consider the domain and range more carefully.

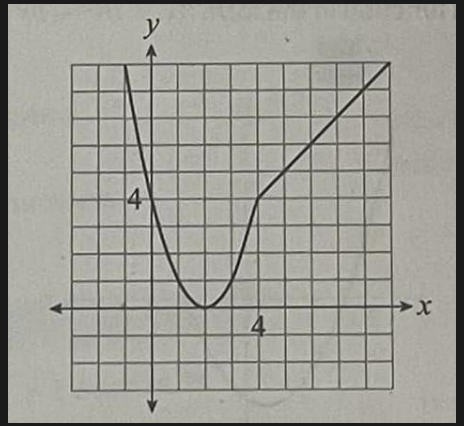
CUBIC FUNCTION



SQUARE ROOT FUNCTION



PIECEWISE FUNCTION



#1 Quadratic Function Rule
#2 Linear Function Rule

The **domain** is the set of all possible inputs to a function, and the **range** is the set of all possible outputs of a function. In a linear function, other than one representing a horizontal line, the domain and range both consist of all real numbers; a line is infinitely long, and every possible x value has a different associated y value. In a nonlinear function, you'll have to consider the domain and range more carefully.

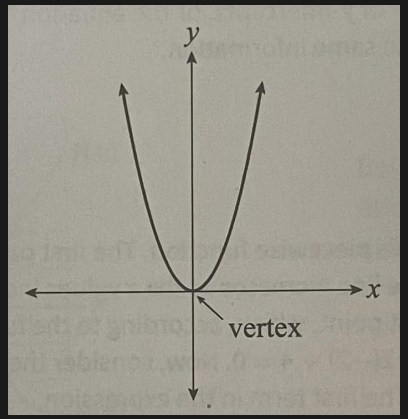
The domain of a function includes every value of x that would result in a real number when plugged into the function. Consider the function $f(x) = \sqrt{x}$. A non-negative number would result in a real number: the positive square root. A negative number would not, since the square root of a negative number is imaginary. Thus, the domain of $f(x) = \sqrt{x}$ is all non-negative numbers. Another common issue to look out for with the domain is division by zero; the domain of $f(x) = \frac{1}{x}$ is all real numbers except 0, as that would make the function undefined.

To determine the range of a function, think about what kind of numbers can be output by the function. Consider again $f(x) = \sqrt{x}$. Since the radical sign by convention refers to the positive square root, the output of this function must be non-negative: either zero or a positive number. So, the range of $f(x) = \sqrt{x}$ is all real numbers greater than or equal to 0.

The SAT may sometimes ask about a function's **minimum** or **maximum**. These terms mean the least and greatest value of the function, respectively. If a function has a minimum, then its range will consist only of numbers greater than or equal to the minimum. Consider the function $f(x) = x^2$. Because squaring any real number results in a non-negative number, this function has a minimum value of 0 at the parabola's vertex, and its range is all real numbers greater than or equal to 0.

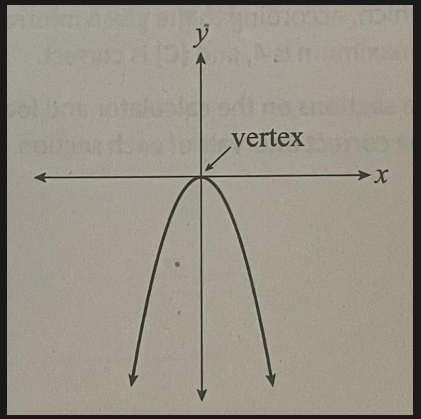
MINIMUM

y= x^2



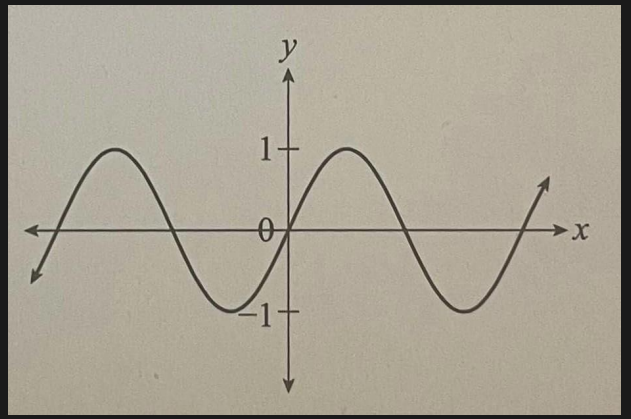
MAXIMUM

y= - (x^2)



PERIODIC WAVE

y = sinx



In many other ways, working with nonlinear functions is no different from what is done with linear functions earlier in the book. For example, evaluating a nonlinear function is just like evaluating a linear one. Take $f(x) = \sqrt{x} + 5$. To evaluate at $x = 4$, it's just a matter of substituting 4 for every x in the equation. $f(4) = \sqrt{4} + 5 = 2 + 5 = 7$.

You need to do this:

If you're given the graph of a function, use it to find out what you need to know about the function, such as its domain, range, minimum, maximum, x- or y-intercepts, or the equation it may be a graph of. If you're not given a graph, use the equation to find out the same information.

Explanation:

Check for a maximum in each part of this piecewise function. The first part of the function, $2x + 4$, is a line segment with a positive slope. Since the line increases as the x values increase, this portion of the function will have its maximum value at its rightmost point, which, according to the function's definition, is at $x = -2$. Plug this into the line formula to get $f(-2) = 2(-2) + 4 = 0$. Now, consider the maximum of the middle piece of the function, which is defined by $-x^2 + 4$. The first term in the expression, $-x^2$, will be negative unless $x = 0$. So, the maximum value this piece of the function can have is $f(0) = -(0)^2 + 4 = 4$. Since the maximum can't be less than 4, eliminate (A) and (B).

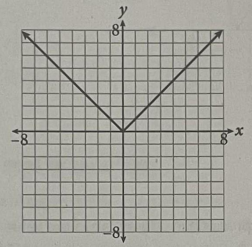
Finally, check the third segment, $-x - 2$. Since this line segment has a negative slope, its greatest value will be at the leftmost point of the segment, which, according to the given information, is at $x = 3$. Plug this in to get $f(3) = -3 - 2 = -5$. So, the function's maximum is 4, and **(C)** is correct.

Alternatively, you could graph the three sections on the calculator and look for the value of the maximum point, making sure to pay attention to only the correct intervals of each section.

SAT Lesson #10
Classwork
Absolute Value & Non-Linear Equations

1

HINT: For Q1, $p(x)$ means the y-value of the function at x.



The figure shows the absolute value function $p(x) = |x|$. Which statement about the function is true?

Ⓐ The range of $p(x)$ is zero.

Ⓑ The domain of $p(x)$ is all positive numbers and zero.

Ⓒ The range of $p(x)$ is all real numbers.

Ⓓ The domain of $p(x)$ is all real numbers.

2

For what values of x is $|2x - 8| + 1$ equal to 3?

(A) -3 and -5

(B) -2 and -5

(C) 2 and 5

(D) 3 and 5

3

$$4x = |9 - 2x|$$

What is the solution to the equation shown?

4

HINT: For Q4, the absolute value of a number is its distance from zero on a number line.

Points c and d on a number line are both 4 units from point a. Which of the following gives the coordinates c and d?

Ⓐ $|x + a| = 4$

Ⓑ $|x - a| = 4$

Ⓒ $|x + 4| = a$

Ⓓ $|x - 4| = a$

5

Which of the following equations is true for some value of x?

Ⓐ $|-x+3|+3=0$

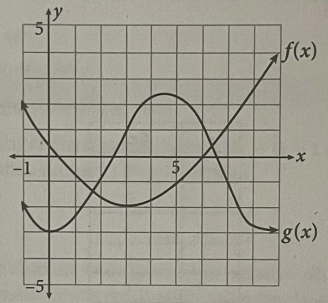
Ⓑ $|x-3|+3=0$

Ⓒ $|x+3|-3=0$

Ⓓ $|x+3|+3=0$

6

HINT: For Q6, remember that $f(x)$ and $g(x)$ are found on the y-axis on the graphs.



In the figure shown, what is the value of $f(3) - g(3)$?

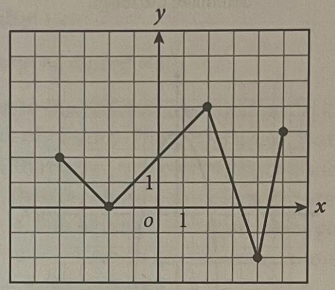
Ⓐ −3

Ⓑ 0

Ⓒ 3

Ⓓ 6

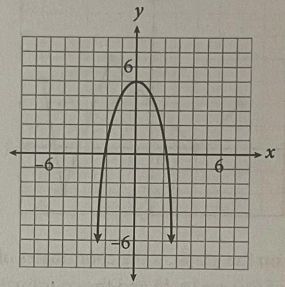
7



Based on the graph, if the coordinates of the maximum of $f(x)$ are (a, b) and the coordinates of the minimum of $f(x)$ are (c, d), what is the value of $a + b + c + d$?

8

HINT: For Q8, the x-values determine the domain, while the y-values determine the range.



The graph of $f(x)$ is above. Which of the following represents the domain and range of the function?

Ⓐ Domain: $f(x) \geq 5$
Range: all real numbers

Ⓑ Domain: $f(x) \leq 5$
Range: all real numbers

Ⓒ Domain: all real numbers
Range: $f(x) \geq 5$

Ⓓ Domain: all real numbers
Range: $f(x) \leq 5$

9

$$f(y) = y^3 - 7y + 5$$

What is $f(5) - f(1)$?

Ⓐ -96

Ⓑ -95

Ⓒ 95

Ⓓ 96

10

x	$f(x)$
-4	4
-1	0
0	-3
3	-9
7	1

The table shows some values for a polynomial defined by the function f. Which of the following is a factor of $f(x)$?

Ⓐ $x - 1$

Ⓑ $x - 3$

Ⓒ $x + 1$

Ⓓ $x + 3$

SAT Classwork #10: Absolute Value & Non-Linear Equations

1)	D
2)	D
3)	3/2
4)	B
5)	C

6)	A
7)	8
8)	D
9)	D
10)	C

SAT Math Module 1

Calculators allowed

35 minutes to complete 22 questions

SAT Math Module 2

Use your calculator

35 minutes to complete 22 questions

A few Test-Taking Strategies

- Prepare in an organized way: Focus on ALGEBRA, GEOMETRY, COORDINATE PLANE, CHARTS & GRAPHS and STATISTICS lessons from Grades 9-10
- Be comfortable with the SAT Level of questions by exposure to as many practice questions as possible. The SAT is a patterned exam.
- Work on Time Management. Be sure to complete "easy to mid" level questions first.
- Some multiple choice questions can be solved by PLUG IN of the answer choices.
- Some multiple choice questions can be simplified by PLUG IN A VALUE for the variable (Plug in "1,2,3,4 or 5")
- ESTIMATE the answer to save procedural time on questions.
- Study and MEMORIZE FORMULAS and SOLUTION METHODS before the exam.
- Look for SHORTCUTS

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